Interactive Session on Methods of Hierarchical and Distributed MPC

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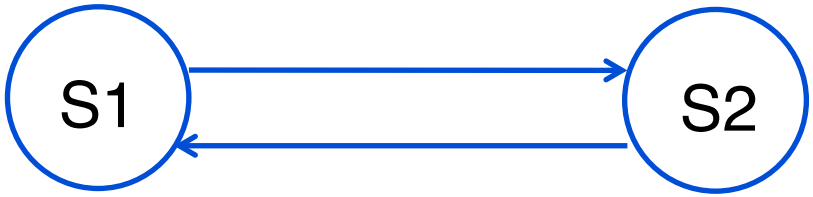
Distributed MPC

Approaches

- Distributed optimization (Leuven)
- Game theory (Winsconsin)
- Robustness (Milano)
- …

A simple example



Subsystem 1
(agent 1)

Subsystem 2
(agent 2)

$$S1:\ x_1(k+1) = A_{11}x_1(k) + A_{12}x_2(k) + B_{11}u_1(k) + B_{12}u_2(k)$$
$$S2:\ x_2(k+1) = A_{21}x_1(k) + A_{22}x_2(k) + B_{21}u_1(k) + B_{22}u_2(k)$$

$$x_1 \in X_1,\ x_2 \in X_2,\ u_1 \in U_1, u_2 \in U_2$$

polytopic sets containing the origin

MPC problem

$$\min_{u_1,u_2} J = \sum_{j=0}^{N-1} [x_1'(k+j)Q_1x_1(k+j) + x_2'(k+j)Q_2x_2(k+j) +$$
$$+ u_1'(k+j)R_1u_1(k+j) + u_2'(k+j)R_2u_2(k+j)] +$$
$$+ x_1'(k+N)P_1x_1(k+N) + x_2'(k+N)P_2x_2(k+N)$$

under the dynamic constraints, the previous state and control constraints, the terminal constraints

$$x_1(k+N) \in X_{f1} \ , \ x_2(k+N) \in X_{f2}$$

and the additional “mixed” (linear) constraints

$$\begin{bmatrix} H_{x1} & H_{x2} & H_{u1} & H_{u2} \end{bmatrix} \begin{bmatrix} x_1(k+j) \\ x_2(k+j) \\ u_1(k+j) \\ u_2(k+j) \end{bmatrix} \leq c \ , \ j = 0,\ldots,N$$

Distributed optimization

The optimization problem

$$\min_{u_1,u_2} J = \sum_{j=0}^{N-1} [x_1'(k+j)Q_1x_1(k+j) + u_1'(k+j)R_1u_1(k+j)] + x_1'(k+N)P_1x_1(k+N) +$$

$$+ \sum_{j=0}^{N-1} [x_2'(k+j)Q_2x_2(k+j) + u_2'(k+j)R_2u_2(k+j)] + x_2'(k+N)P_2x_2(k+N)$$

$$= J_1 + J_2$$

is equivalent to

$$\min_{u_1,u_2} J = \chi_1'(k)\Delta_1\chi_1(k) + \chi_2'(k)\Delta_2\chi_2(k)$$

$$= J_1(\chi_1) + J_2(\chi_2)$$

$$\chi_1(k) = \begin{bmatrix} x_1'(k) & u_1'(k) & \cdots & x_1'(k+N-1) & u_1'(k+N-1) & x_1'(k+N) \end{bmatrix}$$

$$\chi_2(k) = \begin{bmatrix} x_2'(k) & u_2'(k) & \cdots & x_2'(k+N-1) & u_2'(k+N-1) & x_2'(k+N) \end{bmatrix}$$

Distributed optimization

and

$$\min_{u_1,u_2} \; J = \chi_1'(k)\Delta_1\chi_1(k) + \chi_2'(k)\Delta_2\chi_2(k) = J_1\left(\chi_1(k)\right) + J_2\left(\chi_2(k)\right)$$

$$\begin{bmatrix} H_1 & H_2 \end{bmatrix} \begin{bmatrix} \chi_1(k) \\ \chi_2(k) \end{bmatrix} \leq \bar{c}$$

$$\begin{bmatrix} \Gamma_1 & \Gamma_2 \end{bmatrix} \begin{bmatrix} \chi_1(k) \\ \chi_2(k) \end{bmatrix} = 0$$

$$\chi_1(k) \in \Omega_1 \, , \, \chi_2(k) \in \Omega_2$$

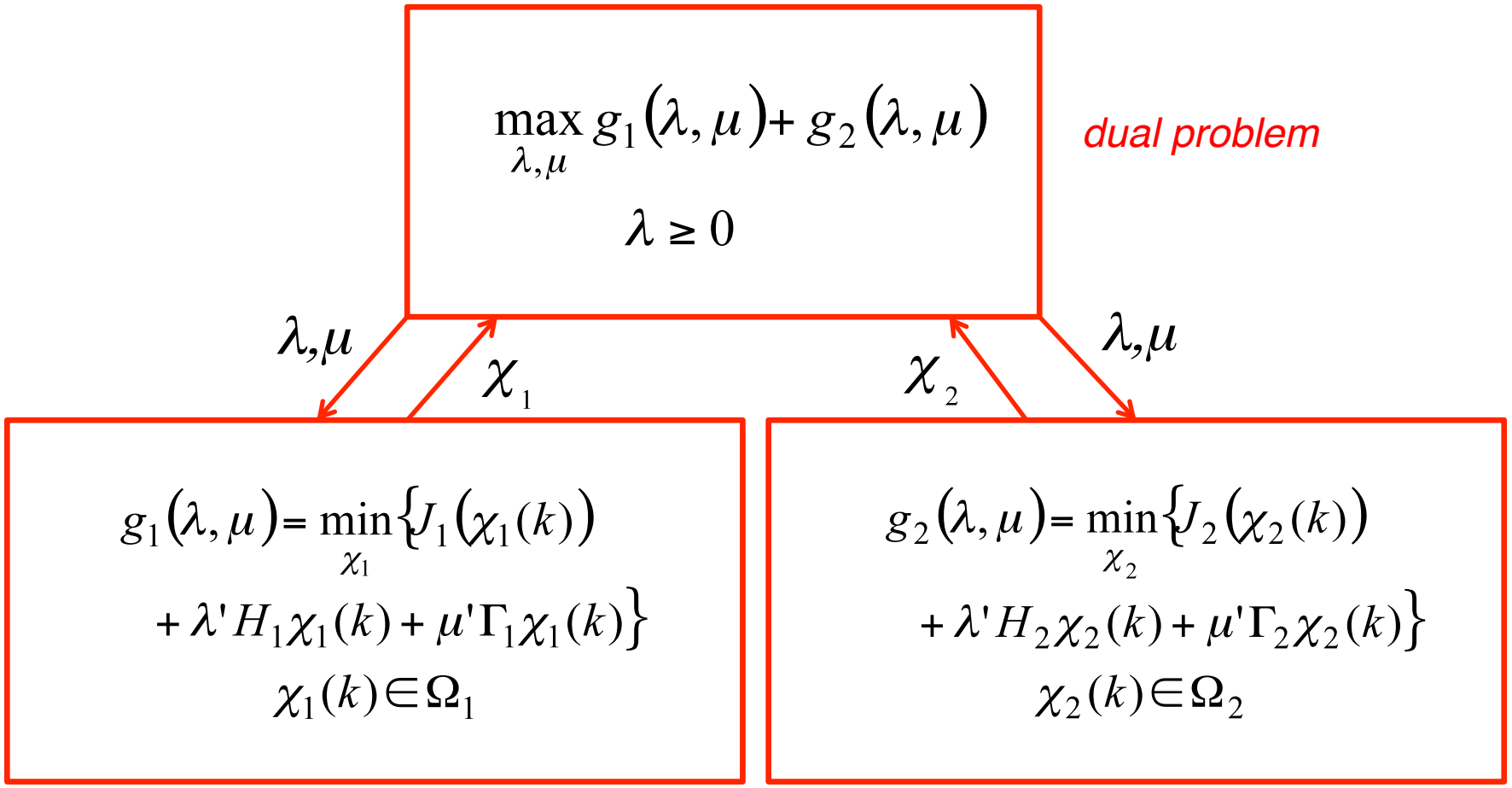
Both the performance index and the constraints are separable. The Lagrangian is also separable

$$L = \chi_1'(k)\Delta_1\chi_1(k) + \chi_2'(k)\Delta_2\chi_2(k) + \lambda'(H_1\chi_1(k) + H_2\chi_2(k) - \bar{c}) + \\ + \mu'(\Gamma_1\chi_1(k) + \Gamma_2\chi_2(k))$$

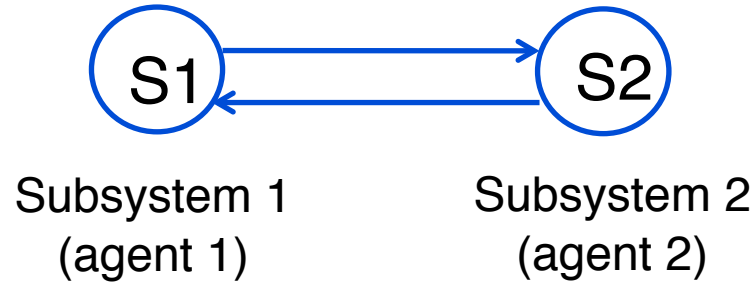
Distributed optimization - 5

Dual decomposition

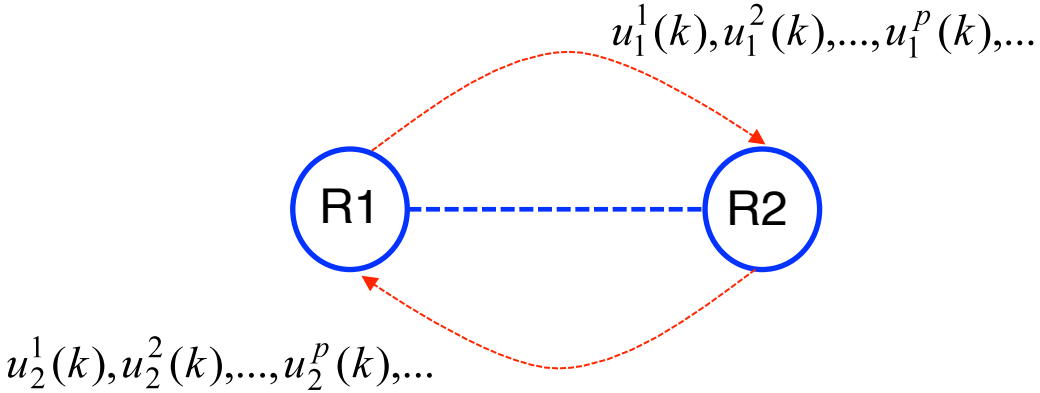
an iterative procedure



Game theory



the two agents play a game, which can be either cooperative or not (Pareto or Nash equilibra are searched for depending on the local or global objective of each agent, or player)



Robust MPC

Subsystem 1
(agent 1)

Subsystem 2
(agent 2)

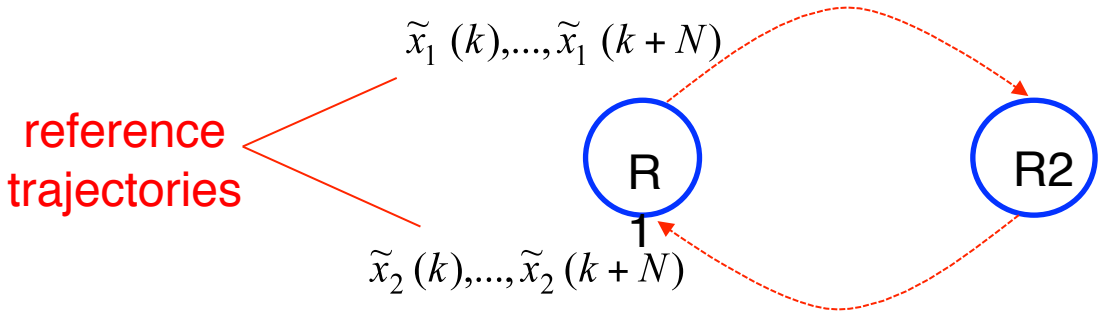
the effect of the other subsystem is viewed as a disturbance to be rejected

$$S1:\ x_1(k+1) = A_{11}x_1(k) + A_{12}x_2(k) + B_{11}u_1(k)$$

$$S2:\ x_2(k+1) = A_{21}x_1(k) + A_{22}x_2(k) + B_{22}u_2(k)$$

w_1

w_2



Distributed control algorithms – *properties*

- *Fully connected*: all-to-all communication. Information is transmitted from any local regulator to all the others
- *Partially connected*: neighbor-to-neighbor communication. Information is transmitted among the local regulators of subsystems with a direct dynamic influence

- *Iterative*: multiple transmissions among local regulators within each sampling time
- *Non iterative:* only one iteration within the sampling time

- *Cooperating*: each local regulator minimizes a global cost function
- *Independent*: each local regulator minimizes a local cost function

Distributed MPC – developments and challenges

- *"Usual stuff"*: output feedback, tracking, nonlinear systems …
- *Reconfigurability*: what happens when new subsystems are added/removed/substituted?
- *Adaptivity*: how to deal with changing operating conditions?
- *Networked control and asyncronous communication*: what is the effect of bandwidth limitation, delays, loss of information …?
- *When distributed MPC*: is it really needed? Is it possible to quantify the loss of performance (also in economic terms)?
- *Software environments*: if SW is not available, distributed MPC will remain an academic game