An overview of distributed model predictive control (MPC)

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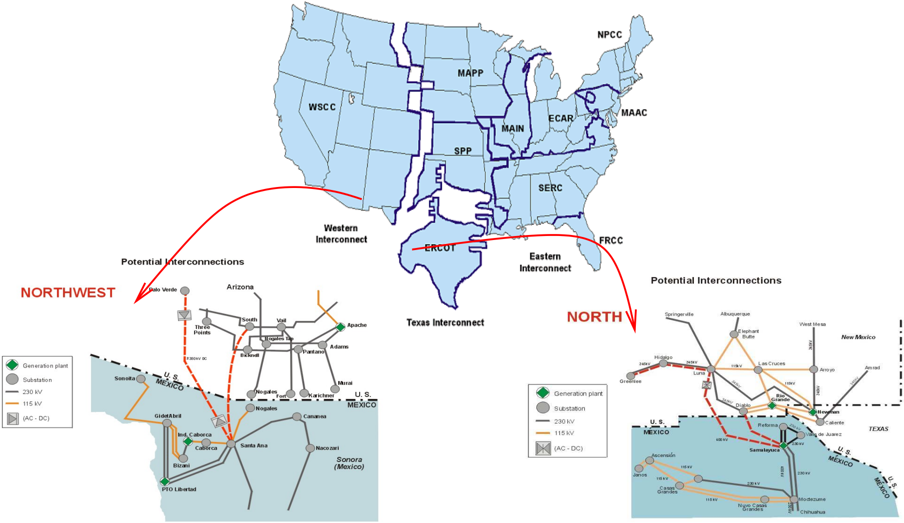
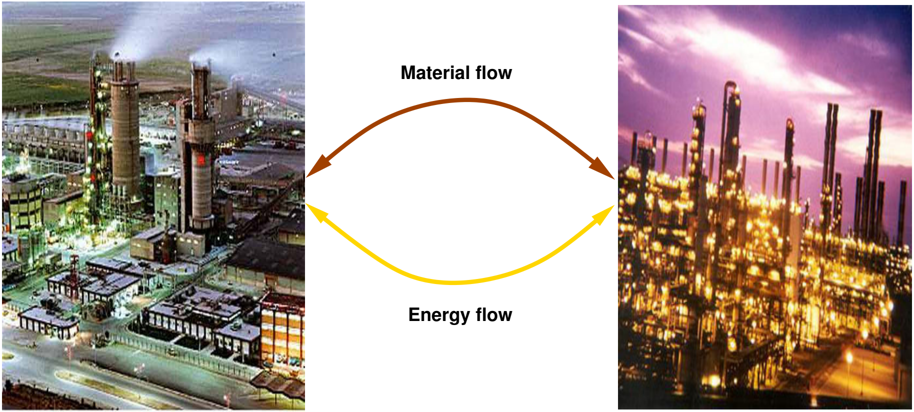
Outline

Electrical power distribution



Chemical plant integration



MPC at the large scale

Decentralized Control

- Most large-scale systems consist of networks of interconnected/interacting subsystems
  - Chemical plants, electrical power grids, water distribution networks, . . .
- Traditional approach: Decentralized control
  - Wealth of literature from the early 1970's on improved decentralized control [a]
  - Well known that poor performance may result if the interconnections are not negligible

[a](Sandell Jr. et al., 1978; Šiljak, 1991; Lunze, 1992)

MPC at the large scale

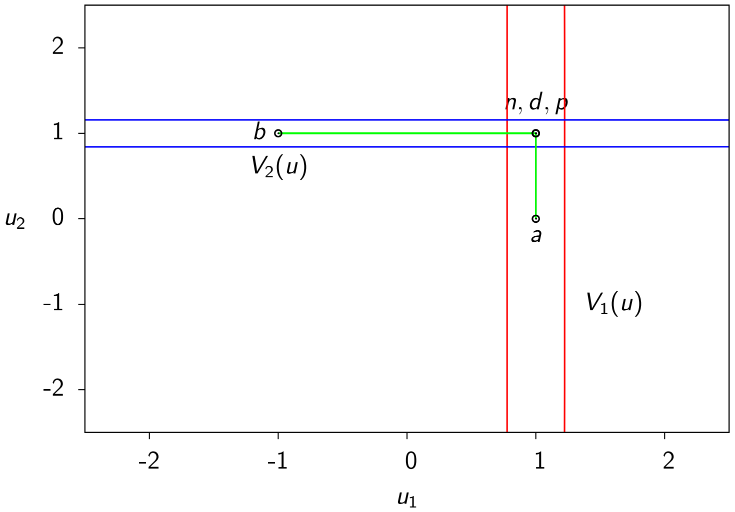
Centralized Control

- Steady increase in available computing power has provided the opportunity for centralized control
- Coordinated control: Distributed optimization to achieve fast solution of centralized control (Necoara et al., 2008; Cheng et al., 2007)
- Most practitioners view centralized control of large, networked systems as impractical and unrealistic
- A divide and conquer strategy is essential for control of large, networked systems (Ho, 2005)
- Centralized control: A benchmark for comparing and assessing distributed controllers

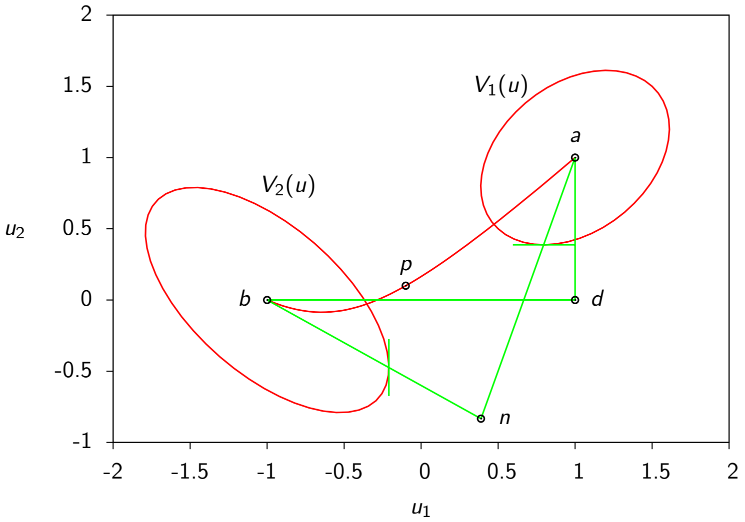
Nomenclature: consider two interacting units

Objective functions	$V_1(u_1, u_2), \ V_2(u_1, u_2)$
and	$V(u_1, u_2) = w_1 V_1(u_1, u_2) + w_2 V_2(u_1, u_2)$
decision variables for units	$u_1 \in \Omega_1, \quad u_2 \in \Omega_2$
Decentralized Control	$\min_{u_1 \in \Omega_1} \widetilde{V}_1(u_1) \quad \min_{u_2 \in \Omega_2} \widetilde{V}_2(u_2)$
Noncooperative Control	$\min_{u_1 \in \Omega_1} V_1(u_1, u_2) \quad \min_{u_2 \in \Omega_2} V_2(u_1, u_2)$
(Nash equilibrium)	
Cooperative Control	$\min_{u_1 \in \Omega_1} V(u_1, u_2) \quad \min_{u_2 \in \Omega_2} V(u_1, u_2)$
(Pareto optimal)	
Centralized Control	$\min_{u_1, u_2 \in \Omega_1 \times \Omega_2} V(u_1, u_2)$
(Pareto optimal)	

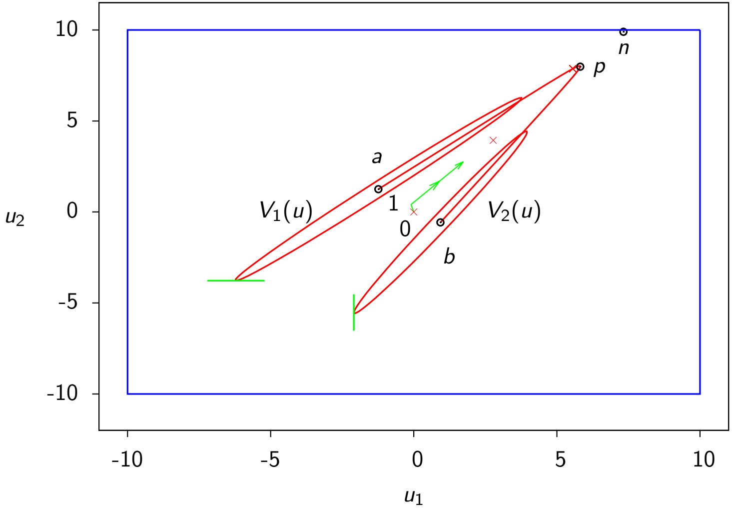
Noninteracting systems



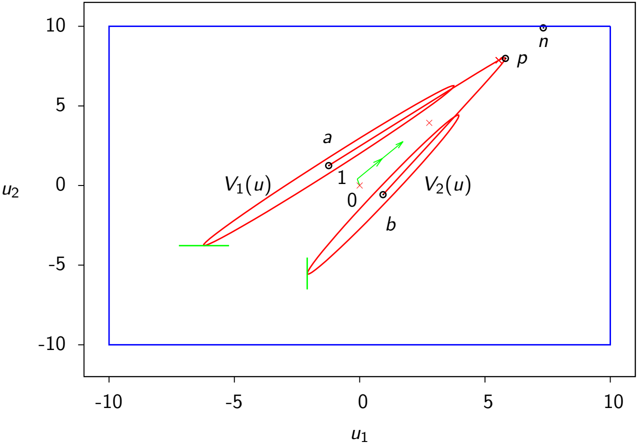
Moderately interacting systems



Geometry of cooperative vs. noncooperative MPC



Plantwide suboptimal MPC



- Early termination of optimization gives suboptimal plantwide feedback
- Use suboptimal MPC theory to prove stability

Plantwide suboptimal MPC

Consider closed-loop system augmented with input trajectory

$$\begin{pmatrix} x^+ \\ \mathbf{u}^+ \end{pmatrix} = \begin{pmatrix} Ax + Bu \\ g(x, \mathbf{u}) \end{pmatrix}$$

- Function $g(\cdot)$ returns suboptimal choice
- Stability of augmented system is established by Lyapunov function

$$a\left|(x,\mathbf{u})\right|^2 \leq V(x,\mathbf{u}) \leq b\left|(x,\mathbf{u})\right|^2$$
$$V(x^+,\mathbf{u}^+) - V(x,\mathbf{u}) \leq -c\left|(x,u)\right|^2$$

- Adding constraint establishes closed-loop stability of the origin for all $\mathbf{u}$[1]

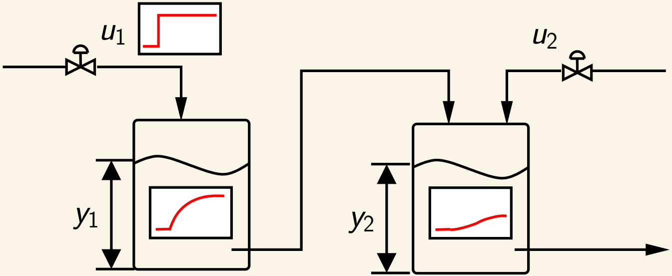
$$|\mathbf{u}| \leq d\,|x| \quad x \in \mathbb{B}_r, r > 0$$

- Cooperative optimization satisfies these properties for plantwide objective function $V(x, \mathbf{u})$

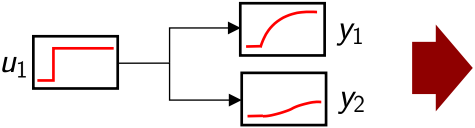
[1](Rawlings and Mayne, 2009, pp.418-420)

Modeling

Plantwide step response



- Interaction models found by decentralized identification[2]



$$x_{11}^{+} = A_{11}x_{11} + B_{11}u_1$$

$$x_{21}^{+} = A_{21}x_{21} + B_{21}u_1$$

[2]Gudi and Rawlings (2006)

Modeling

Consider the linearized physical model

$$x^+ = Ax + B_1 u_1 + B_2 u_2 \qquad y_1 = C_1 x, \quad y_2 = C_2 x$$

- Kalman canonical form of the triple (A, B_j, C_i)

$$\begin{bmatrix} z_{ij}^{oc} \\ z_{ij}^{\bar{o}c} \\ z_{ij}^{o\bar{c}} \\ z_{ij}^{\bar{o}\bar{c}} \end{bmatrix}^+ = \begin{bmatrix} A_{ij}^{oc} & 0 & A_{ij}^{oc\bar{c}} & 0 \\ A_{ij}^{\bar{o}oc} & A_{ij}^{\bar{o}c} & A_{ij}^{\bar{o}co\bar{c}} & A_{ij}^{\bar{o}c\bar{c}} \\ 0 & 0 & A_{ij}^{o\bar{c}} & 0 \\ 0 & 0 & A_{ij}^{\bar{o}\bar{c}o} & A_{ij}^{\bar{o}\bar{c}} \end{bmatrix} \begin{bmatrix} z_{ij}^{oc} \\ z_{ij}^{\bar{o}c} \\ z_{ij}^{o\bar{c}} \\ z_{ij}^{\bar{o}\bar{c}} \end{bmatrix} + \begin{bmatrix} B_{ij}^{oc} \\ B_{ij}^{\bar{o}c} \\ 0 \\ 0 \end{bmatrix} u_j$$

$$y_{ij} = \begin{bmatrix} C_{ij}^{oc} & 0 & C_{ij}^{o\bar{c}} & 0 \end{bmatrix} \begin{bmatrix} z_{ij}^{oc} \\ z_{ij}^{\bar{o}c} \\ z_{ij}^{o\bar{c}} \\ z_{ij}^{\bar{o}\bar{c}} \end{bmatrix} \qquad y_i = \sum_j y_{ij}$$

- Interaction models

$$A_{ij} \leftarrow A_{ij}^{oc} \quad B_{ij} \leftarrow B_{ij}^{oc} \quad C_{ij} \leftarrow C_{ij}^{oc} \quad x_{ij} \leftarrow z_{ij}^{oc}$$

Unstable modes

For unstable systems, we zero the unstable modes with terminal constraints.

- For subsystem 1

$$S_{11}^{u\,\prime} x_{11}(N) = 0 \qquad S_{21}^{u\,\prime} x_{21}(N) = 0$$

- To ensure terminal constraint feasibility for all x, we require $(\underline{A}_1, \underline{B}_1)$ stabilizable

$$\underline{A}_1 = \begin{bmatrix} A_{11} & \\ & A_{21} \end{bmatrix} \qquad \underline{B}_1 = \begin{bmatrix} B_{11} \\ B_{21} \end{bmatrix}$$

- For output feedback, we require (A_1, C_1) detectable

$$A_1 = \begin{bmatrix} A_{11} & \\ & A_{12} \end{bmatrix} \quad C_1 = \begin{bmatrix} C_{11} & C_{12} \end{bmatrix}$$

- Similar requirements for other subsystem

Output feedback

Consider augmented system perturbed by stable estimator

$$\begin{pmatrix} \hat{x}^+ \\ \mathbf{u}^+ \\ e^+ \end{pmatrix} = \begin{pmatrix} A\hat{x} + B\mathbf{u} + Le \\ g(\hat{x}, \mathbf{u}, e) \\ A_L e \end{pmatrix}$$

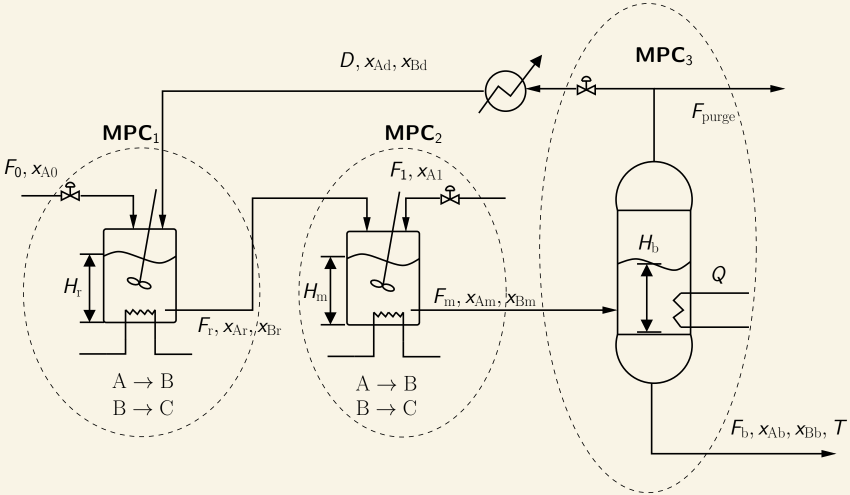
- Stable estimator error implies Lyapunov function

$$\begin{aligned} \bar{a}\,|e| \leq & J(e) \leq \bar{b}\,|e| \\ J(e^+) - & J(e) \leq -\bar{c}\,|e| \end{aligned}$$

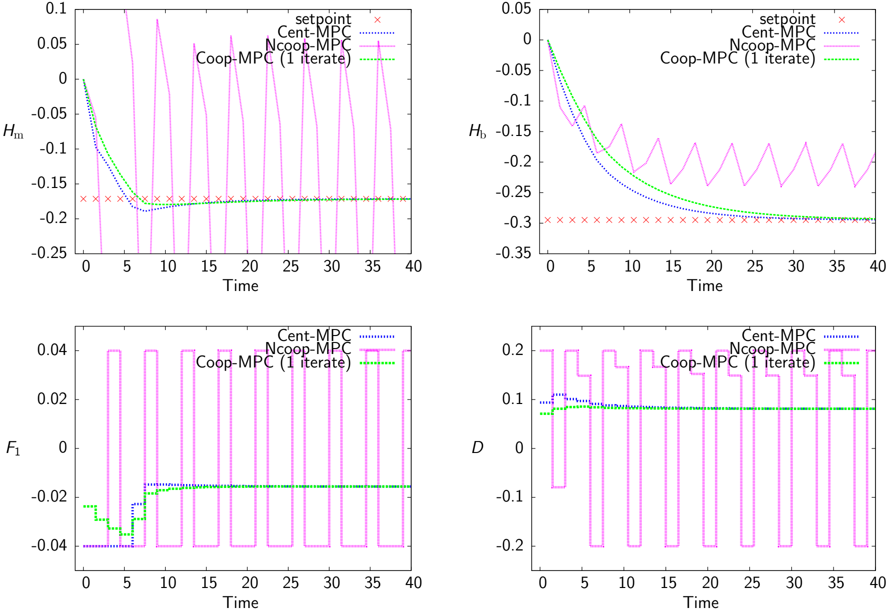
- Stability of perturbed system established by Lyapunov function

$$W(\hat{x}, \mathbf{u}, e) = V(\hat{x}, \mathbf{u}) + J(e)$$

Two reactors with separation and recycle



Two reactors with separation and recycle

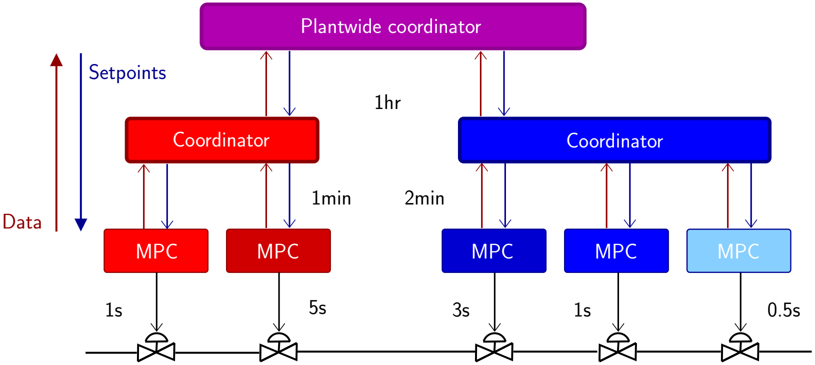


Two reactors with separation and recycle

Performance comparison

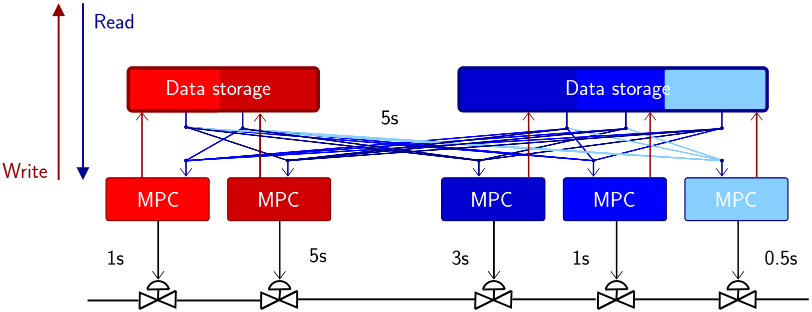
	Cost ($\times 10^{-2}$)	Performance loss
Centralized MPC	1.75	0
Decentralized MPC	∞	∞
Noncooperative MPC	∞	∞
Cooperative MPC (1 iterate)	2.2	25.7%
Cooperative MPC (10 iterates)	1.84	5%

Traditional hierarchical MPC



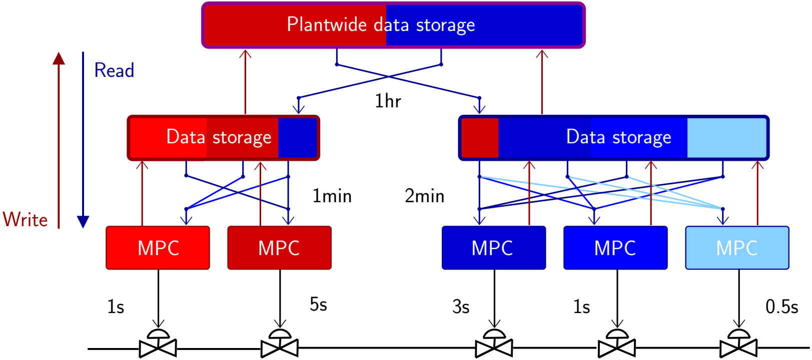
- Multiple dynamical time scales in plant
- Data and setpoints are exchanged on chosen scale
- Optimization performed at each layer

Cooperative MPC data exchange



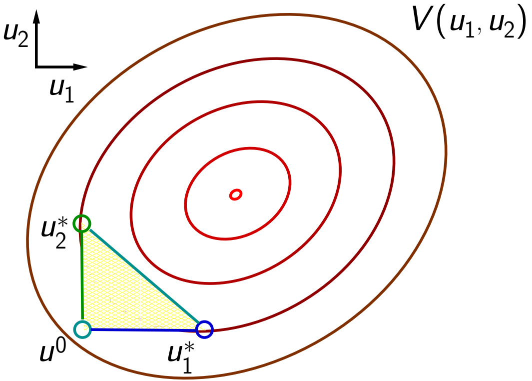
- All data exchanged plantwide
- Data exchange at each controller execution

Cooperative hierarchical MPC



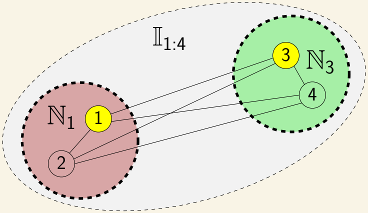
- Optimization at MPC layer only
- Only subset of data exchanged plantwide
- Data exchanged at chosen time scale

Motivating the hierarchical optimization



- Any point in the triangle decreases the cost of V

Hierarchical optimization



Consider the optimization

$$\min_u V(u_1, u_2, u_3, u_4)$$

We group the variables into two neighborhoods

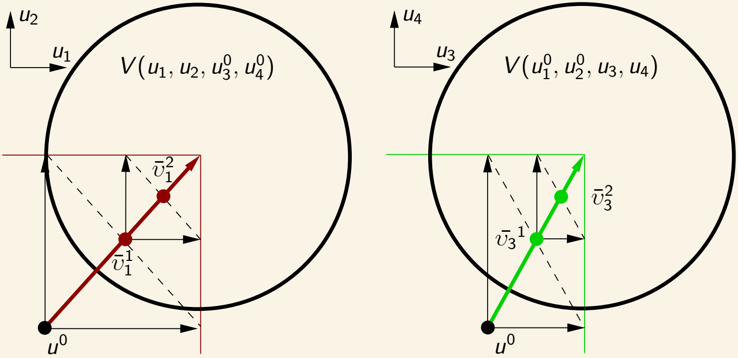
- $\mathbb{N}_1 = \{1, 2\}$ and $\mathbb{N}_2 = \{3, 4\}$

We solve the optimization in a distributed fashion

- suboptimizations utilize the latest iterate only from variables in their neighborhood

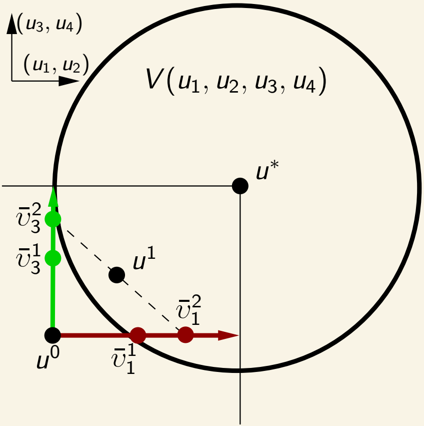
Hierarchical optimization

Suboptimizations



Hierarchical optimization

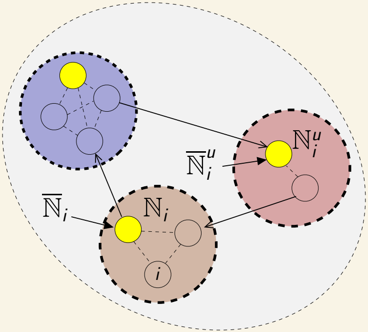
Overall



Performance comparison

	Cost	Performance loss
Centralized	0.95	-
Cooperative (1 iterate)	1.60	68%
$N_s = 1$	1.633	71%
$N_s = 2$	1.646	73%
$N_s = 5$	1.661	75%
$N_s = 10$	1.669	76%
$N_s = 25$	1.670	76%
$N_s = 50$	1.670	76%

Reducing communication



We define a leader in each neighborhood and a graph between the leaders

Reducing communication

We define the state propagation in the following way

$$x_i(k) = \bar{A}_{ii}^k x_i(0) + \sum_{\tau=0}^{k-1} \sum_{j \in \mathbb{N}_i} \bar{A}_{ii}^{k-\tau-1} \bar{B}_{ij} u_j(\tau)$$
$$+ \sum_{\tau=0}^{k-1} \sum_{l \in \mathbb{L}} \sum_{s \in \mathbb{I}_{1:M} \setminus l} \bar{A}_{is}^{[k-\tau-1]} \bar{A}_{sl} \alpha_l(\tau)$$

such that

$$\alpha_i^+ = \bar{A}_{ii} \alpha_i + \sum_{j \in \mathbb{N}_i} \bar{B}_{ij} u_j$$

- α is defined only for the leaders
- Computation requires only information from within the neighborhood and from other leaders

Nonlinear Distributed MPC

We assume the model is of the form

$$\begin{aligned}\frac{dx_1}{dt} &= f_1(x_1, x_2, u_1, u_2)\\ y_1 &= C_1 x_1\\ \frac{dx_2}{dt} &= f_2(x_1, x_2, u_1, u_2)\\ y_2 &= C_2 x_2\end{aligned}$$

Given these physical system models of the subsystems, the overall plant model is

$$\begin{aligned}\frac{dx}{dt} &= f(x, u)\\ y &= Cx\end{aligned}$$

in which

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad f = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \qquad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \qquad C = \begin{bmatrix} C_1 & \\ & C_2 \end{bmatrix}$$

Nonconvexity

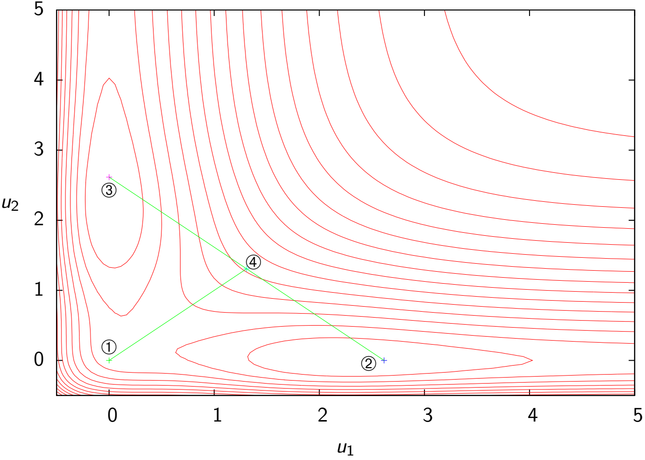


Figure: Cost contours for a two-player, nonconvex game; cost *increases* for the convex combination of the two players' optimal points.

Requirements for distributed, nonlinear control

- Must handle nonconvex objectives
- Two criteria in design:
  1. the optimizers should *not* rely on a central coordinator
  2. the exchange of information between the subsystems and the iteration of the subsystem optimizations should be able to terminate before convergence without compromising closed-loop properties.

Distributed nonconvex optimization

- Consider the optimization

$$\min_u V(u) \quad \text{s.t.} \quad u \in \mathbb{U}$$

- We require approximate solutions to the following suboptimizations at iterate $p \geq 0$ for all $i \in \mathbb{I}_{1:M}$

$$\overline{u}_i^p = \arg \min_{u_i \in \mathbb{U}_i} V(u_i, u_{-i}^p)$$

in which $u_{-i} = (u_1, \ldots, u_{i-1}, u_{i+1}, \ldots, u_M)$.

- Define the step $\upsilon_i^p = \overline{u}_i^p - u_i^p$.

Algorithm

- To choose the stepsize α_i^p, each suboptimizer initializes the stepsize[3] with $\overline{\alpha}_i$

$$V(u^p) - V(u_i^p + \alpha_i^p \upsilon_i^p, u_{-i}^p) \geq -\sigma \alpha_i^p \nabla_i V(u^p)' \upsilon_i^p$$

  in which $\sigma \in (0, 1)$.

- After all suboptimizers finish the backtracking process, they exchange steps. Each suboptimizer forms a candidate step

$$u_i^{p+1} = u_i^p + w_i \alpha_i^p \upsilon_i^p \quad \forall i \in \mathbb{I}_{1:M}$$

[3] Armijo rule: (Bertsekas, 1999, p.230)

Algorithm

- Check the following inequality, which tests if $V(u^p)$ is convex-like

$$V(u^{p+1}) \leq \sum_{i \in \mathbb{I}_{1:M}} w_i V(u_i^p + \alpha_i^p \upsilon_i^p, u_{-i}^p) \tag{1}$$

  in which $\sum_{i \in \mathbb{I}_{1:M}} w_i = 1$ and $w_i > 0$ for all $i \in \mathbb{I}_{1:M}$.
- If the condition above is not satisfied, then we find the direction with the worst cost improvement

$$i_{\max} = \arg\max_i \{V(u_i^p + \alpha_i^p \upsilon_i^p, u_{-i}^p)\}$$

  and eliminate this direction by setting $w_{i_{\max}}$ to zero and repartitioning the remaining w_i so that they sum to 1.
- *At worst, condition* (1) *is satisfied with one direction only.*

Distributed nonconvex optimization — Properties

Lemma (Feasibility)

Given a feasible initial condition, the iterates u^p *are feasible for all* $p \geq 0$.

Lemma (Objective decrease)

The objective function decreases at every iterate, that is,
$V(u^{p+1}) \leq V(u^p)$.

Lemma (Convergence)

Every accumulation point of the sequence $\{u^p\}$ *is stationary.*

Distributed nonconvex optimization

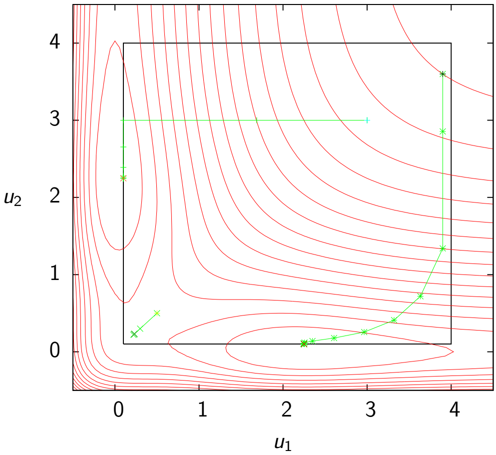


Figure: Nonconvex function optimized with Distributed nonconvex optimization algorithm

A nonlinear example

- Consider the unstable nonlinear system

$$x_1^+ = x_1^2 + x_2 + u_1^3 + u_2$$
$$x_2^+ = x_1 + x_2^2 + u_1 + u_2^3$$

with initial condition $(x_1, x_2) = (3, -3)$.

- For this example, we use the stage cost

$$\ell_1(x_1, u_1) = \frac{1}{2}(x_1' Q_1 x_1 + u_1' R_1 u_1)$$
$$\ell_2(x_2, u_2) = \frac{1}{2}(x_2' Q_2 x_2 + u_2' R_2 u_2)$$

- For the simulation we choose the parameters

$$Q = I \quad R = I \quad N = 2 \quad \overline{p} = 3 \quad \mathbb{U}_i = [-2.5, 2.5] \quad \forall i \in \mathbb{I}_{1:2}$$

Distributed nonlinear cooperative control

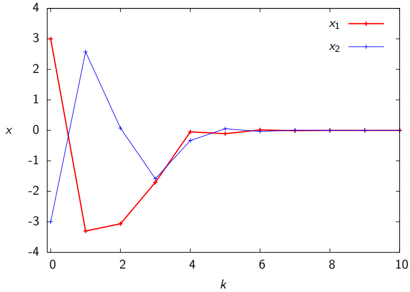


Figure: State trajectory ($\overline{p} = 3$)

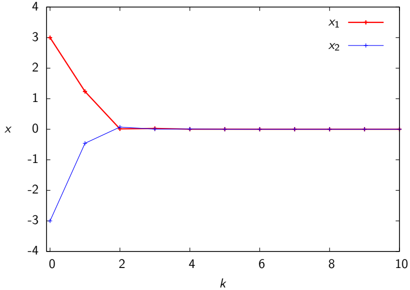


Figure: Centralized state trajectory ($\overline{p} = 10$)

Distributed nonlinear cooperative control

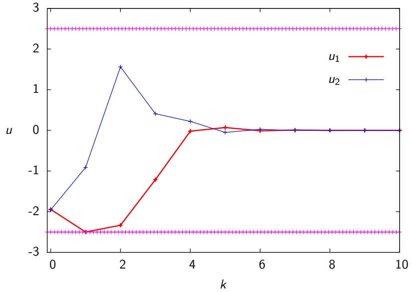


Figure: Input trajectory ($\overline{p} = 3$)

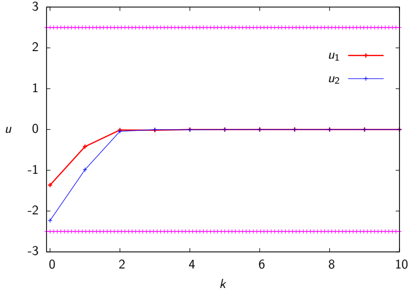


Figure: Centralized input trajectory ($\overline{p} = 10$)

Distributed nonlinear cooperative control

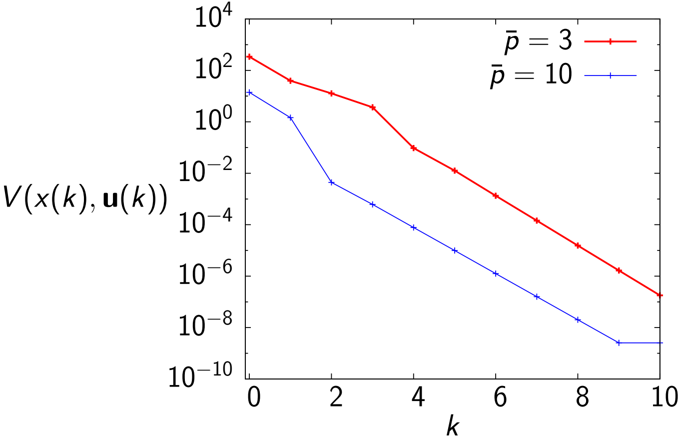


Figure: Open-loop cost to go versus time on the closed-loop trajectory for different numbers of iterations.

Distributed nonlinear cooperative control

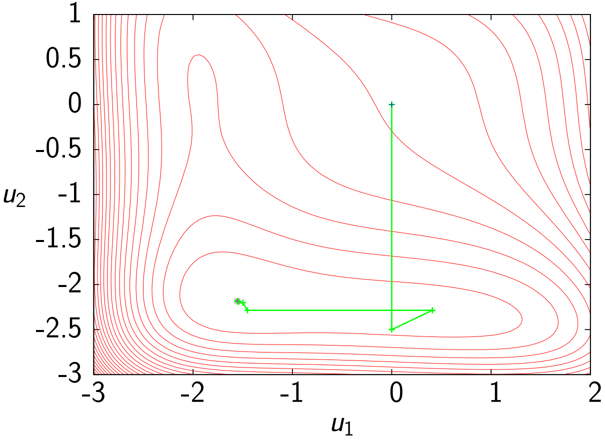


Figure: Contours of V with $N = 1$ for $k = 0$ with $(x_1(0), x_2(0)) = (3, -3)$. Iterations of the subsystem controllers with initial condition $(u_1^0, u_2^0) = (0, 0)$.

Why study robustness of *suboptimal* MPC?

- *Cooperative*, distributed MPC is a special case of *suboptimal* MPC. Anything we establish about suboptimal MPC can be applied to cooperative, distributed MPC (and optimal MPC!)
- Suboptimal MPC has an interesting feature: a nonunique, point-to-set control law $u \in \kappa_N(x)$.
- *Optimal* solution of nonconvex

$$\mathbb{P}_N(x): \qquad \min_{\mathbf{u} \in \mathcal{U}_N} V_N(x, \mathbf{u})$$

  cannot be computed online for *any* nonlinear model. Practitioners implement only suboptimal MPC.
- We should know something about its inherent robustness properties.[4]

[4] Pannocchia et al. (2011)

For suboptimal MPC; again, the basic MPC setup

- The system model

$$x^+ = f(x, u) \tag{2}$$

- State and input constraints

$$x(k) \in \mathbb{X}, \qquad u(k) \in \mathbb{U} \qquad \text{for all } k \in \mathbb{I}_{\geq 0}$$

- Terminal constraint (and penalty)

$$\phi(N; x, \mathbf{u}) \in \mathbb{X}_f \subseteq \mathbb{X}$$

Cost function and control problem

- For any state $x \in \mathbb{R}^n$ and input sequence $\mathbf{u} \in \mathbb{U}^N$, we define

$$V_N(x, \mathbf{u}) = \sum_{k=0}^{N-1} \ell(\phi(k; x, \mathbf{u}), u(k)) + V_f(\phi(N; x, \mathbf{u}))$$

- $\ell(x, u)$ is the stage cost; $V_f(x(N))$ is the terminal cost
- Consider the finite horizon optimal control problem

$$\mathbb{P}_N(x) : \qquad \min_{\mathbf{u} \in \mathcal{U}_N} V_N(x, \mathbf{u})$$

Suboptimal MPC

- Rather then solving $\mathbb{P}_N(x)$ *exactly*, we consider using any (unspecified) suboptimal algorithm having the following properties.
- Let $\mathbf{u} \in \mathcal{U}_N(x)$ denote the (suboptimal) control sequence for the initial state x, and let $\tilde{\mathbf{u}}$ denote a *warm start* for the successor initial state $x^+ = f(x, u(0; x))$, obtained from $(x, \mathbf{u})$ by

$$\tilde{\mathbf{u}} := \{u(1; x), u(2; x), \ldots, u(N-1; x), u_+\} \tag{3}$$

- $u_+ \in \mathbb{U}$ is any input that satisfies the invariance condition in the terminal region

Suboptimal MPC

- The warm start satisfies $\tilde{\mathbf{u}} \in \mathcal{U}_N(x^+)$.
- The suboptimal input sequence for any given $x^+ \in \mathcal{X}_N$ is defined as *any* $\mathbf{u}^+ \in \mathbb{U}^N$ that satisfies:

$$\mathbf{u}^+ \in \mathcal{U}_N(x^+) \tag{4a}$$

$$V_N(x^+, \mathbf{u}^+) \leq V_N(x^+, \tilde{\mathbf{u}}) \tag{4b}$$

$$V_N(x^+, \mathbf{u}^+) \leq V_f(x^+) \qquad \text{when } x^+ \in r\mathbb{B} \tag{4c}$$

  in which r is a positive scalar sufficiently small that $r\mathbb{B} \subseteq \mathbb{X}_f$.
- Notice that constraint (4c) is required to hold only if $x^+ \in r\mathbb{B}$, and it implies that $|\mathbf{u}^+| \to 0$ as $|x^+| \to 0$.
- Condition (4b) ensures that the computed suboptimal cost is no larger than that of the warm start.

Inherent robustness of the suboptimal controller

- Consider a process disturbance d, $x^+ = f(x, \kappa(x)) + d$
- A measurement disturbance $x_m = x + e$
- Nominal controller with disturbance

$$\begin{aligned} x^+ &\in f(x, \kappa_N(x_m)) + d \\ x^+ &\in f(x, \kappa_N(x + e)) + d \\ x^+ &\in F_{ed}(x) \end{aligned} \tag{5}$$

Robust stability; is the system $x^+ \in F_{ed}(x)$ input-to-state stable considering (d, e) as the input.

Robust exponential stability of suboptimal MPC

Definition (SRES)

The origin of the closed-loop system (5) is *strongly robustly exponentially stable* (SRES) on a compact set $\mathcal{C} \subset \mathcal{X}_N$, $0 \in \text{int}(\mathcal{C})$, if there exist scalars $b > 0$ and $0 < \lambda < 1$ such that the following property holds: Given any $\epsilon > 0$, there exists $\delta > 0$ such that for all sequences $\{d(k)\}$ and $\{e(k)\}$ satisfying

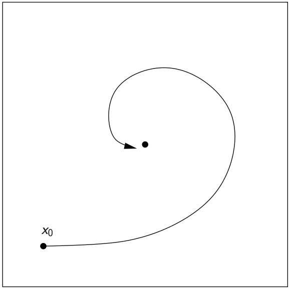
$$|d(k)| \leq \delta \text{ and } |e(k)| \leq \delta \quad \text{for all } k \in \mathbb{I}_{\geq 0},$$

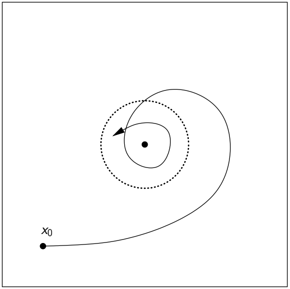
and all $x \in \mathcal{C}$, we have that

$$x_m(k) = x(k) + e(k) \in \mathcal{X}_N, \; x(k) \in \mathcal{X}_N, \; \text{for all } k \in \mathbb{I}_{\geq 0}, \tag{6a}$$

$$|\phi_{ed}(k; x)| \leq b\lambda^k |x| + \epsilon, \; \text{for all } k \in \mathbb{I}_{\geq 0}. \tag{6b}$$

Behavior with and without disturbances





Nominal System

$$x^+ = f(x, u)$$
$$u = \kappa_N(x)$$

System with Disturbance

$$x^+ = f(x, u) + d$$
$$u = \kappa_N(x + e)$$

d is the process disturbance

e is the measurement disturbance

Main results

Theorem (SRES of suboptimal MPC (Pannocchia et al., 2011))

Under standard MPC assumptions, the origin of the perturbed closed-loop system

$$x^+ \in F_{ed}(x)$$

is SRES on $\mathcal{C}_\rho$.

This result applies also to distributed, cooperative MPC.
See also Pannocchia talk on Wednesday, 14:30, WEB07.4.

Conclusions

Cooperative MPC theory maturing[a]

[a] Stewart et al. (2010); Maestre et al. (2011)

- Avoids coordination layer
- Satisfies hard input constraints
- Provides nominal stability for plants with even strongly interacting subsystems
- Retains closed-loop stability for early iteration termination
- Converges with iteration to Pareto optimal (centralized) control
- Remains stable under perturbations

Future directions

Lots to do!

- Applications in which players *compete* as well as cooperate
- Framework(s) for decomposing large-scale systems
- Modeling versus performance tradeoffs poorly understood
- Unstable systems and coupled constraints difficult to handle (supply chain)
- Distributed state estimation has received less attention than control (Farina et al., 2010a,b)
- Applications exposing limitations of current approaches (De Schutter and Scattolini, 2011; Tarau et al., 2011; Baskar et al., 2011)

Further reading I

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Further reading III

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