DISA

Distributed Model Predictive Control Based on Game Theory

Departamento de Ingeniería de Sistemas y Automática
Universidad de Sevilla

J.M. Maestre, D. Limón and
D. Muñoz de la Peña

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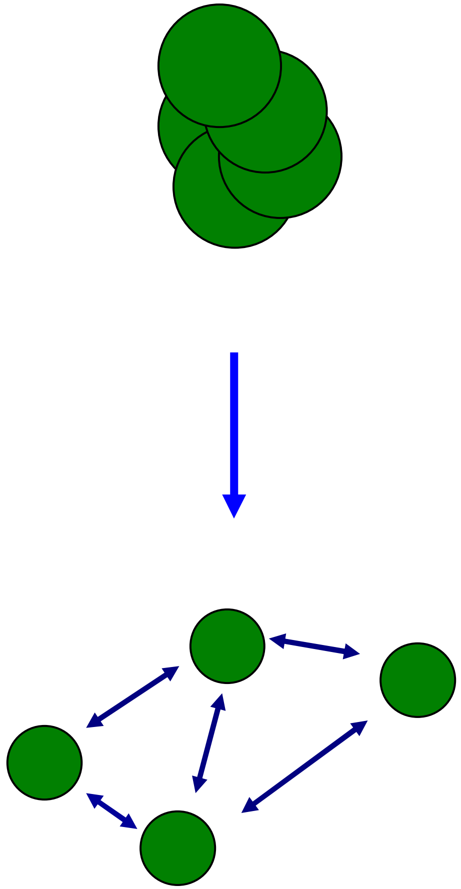
Outline

- Introduction
- DMPC scheme for Two Agents
- DMPC scheme for Multiple Agents
- Conclusions and Further Research

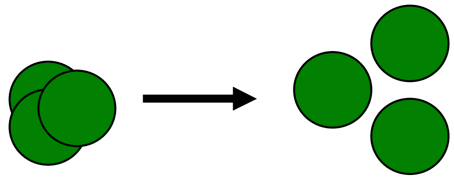
Introduction

- Standard centralized control systems
  - Single controller
  - Flawless communication
- Implementation problems
  - System-wide model
  - Computation time
  - Large scale systems
    - Transportation networks
  - Communication constraints
  - Concerns about privacy
    - Supply chains
- Distributed control
  - Multiple controllers/agents
  - Communication
  - Partial system knowledge

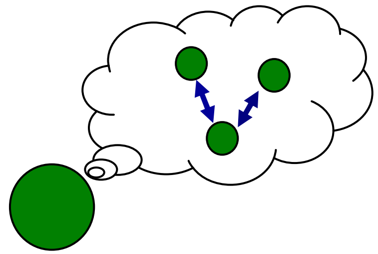


Introduction

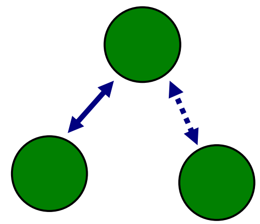
- Many control schemes have been proposed with differences on:
  - System decomposition
    - Systems coupled through the inputs
    - 2 and N subsystems



  - Information available
    - Local model and measurements



  - Communicational constraints
    - Agent to agent communication
    - Low communicational burden

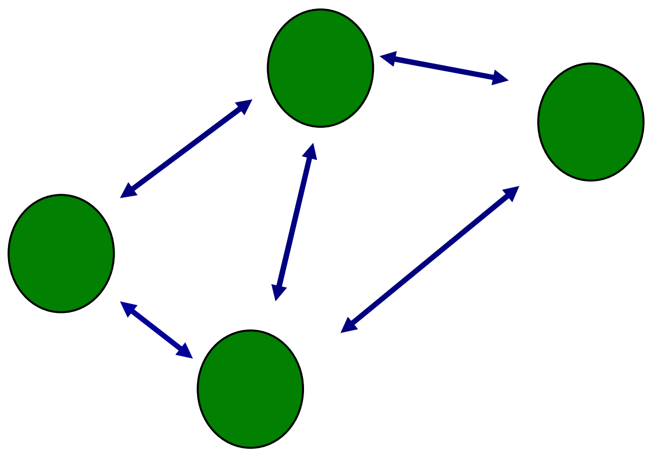


Introduction

Game theory

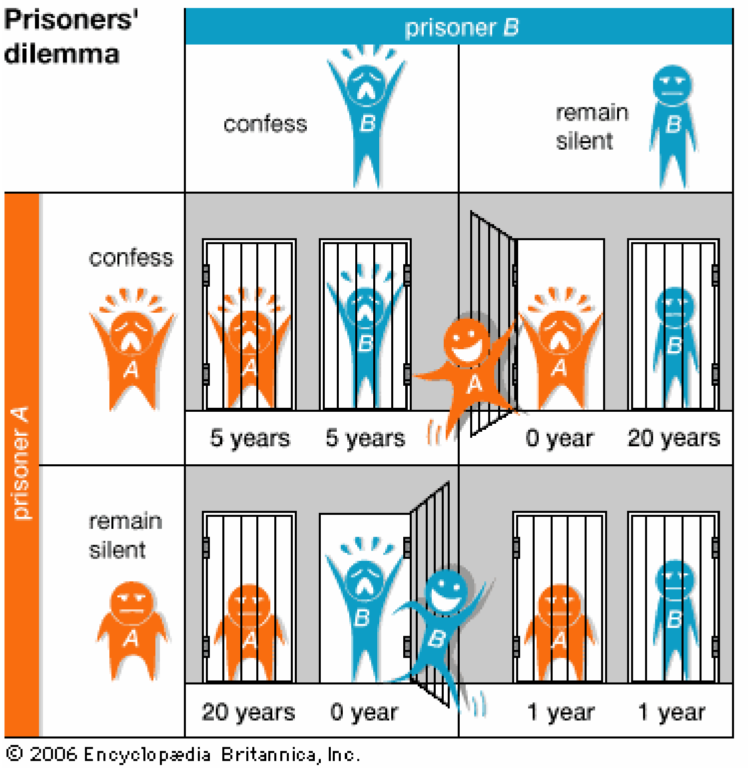
"Game theory is a mathematical field that studies the process of interactive decision making, that is, situations in which there are several entities, namely players or agents, whose individual decisions determine jointly the final outcome."

- Cooperative games
- Bargaining processes
- Coalitions



Introduction

- Cooperative games
  - Prisioners' dilemma

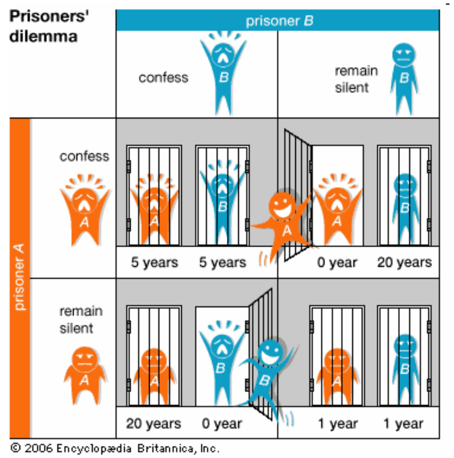


What would ***you*** do?

Cooperative game theory assume all the players "*cooperate*"

Introduction

- Cooperative games
  - Prisioners' dilemma



Prisioner A point of view

	B remains silent	B confesses
A remains silent	1 year	20 year
A confesses	goes free	5 years

Prisioner B point of view

	B remains silent	B confesses
A remains silent	1 year	goes free
A confesses	20 years	5 years

"Greater good"

	B remains silent	B confesses
A remains silent	**2 years**	20 year
A confesses	20 years	10 years

(global cost function)
(global knowledge)
(communicate)

DMPC scheme for two agents

- Assumptions
  - There is no coupling between the states of the agents, only in the actuation

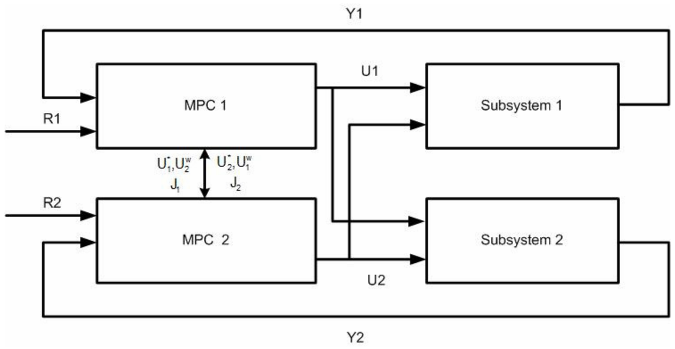
$$x_1(t+1) = A_1 x_1(t) + B_{11} u_1(t) + B_{12} u_2(t)$$
$$x_2(t+1) = A_2 x_2(t) + B_{21} u_1(t) + B_{22} u_2(t)$$

$$x_i \in \mathcal{X}_i,\, u_i \in \mathcal{U}_i,\; i = 1,2$$

  - Input and state constraints

$$J_1(x_1, U_1, U_2) = \sum_{k=0}^{N-1} L_1(x_{1,k}, u_{1,k}) + F_1(x_{1,N})$$
$$J_2(x_2, U_2, U_1) = \sum_{k=0}^{N-1} L_2(x_{2,k}, u_{2,k}) + F_2(x_{2,N})$$

  - Each agent has local information about the state and model

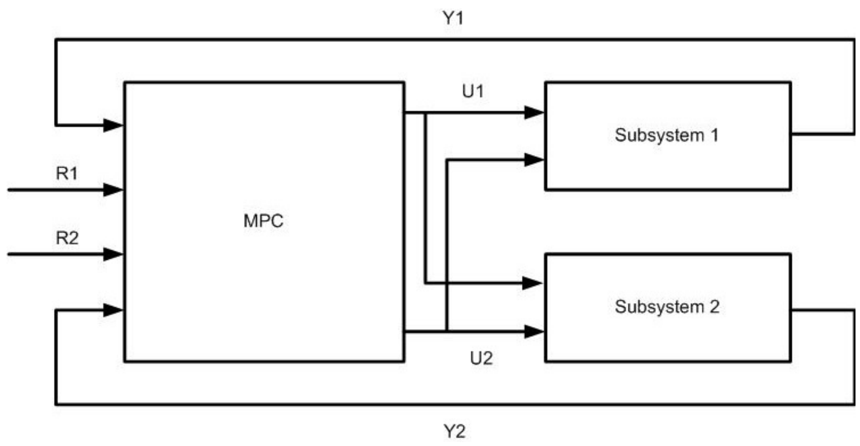


  - Agents optimize according to a local cost function

DMPC scheme for two agents

- Centralized MPC

$$\min_{U_1,U_2} \quad J(U_1, U_2, x_1, x_2) = J_1 + J_2$$



Definition of the global cost function (“greater good”)

DMPC scheme for two agents

- Algorithm
  - Each agent receives its state info x_i
  - Each agent evaluates U^s shifting the last decided input trajectory U^d

$$U_1^d(t) = \begin{bmatrix} u_{1,0}^d \\ u_{1,1}^d \\ \vdots \\ u_{1,N-1}^d \end{bmatrix}, \; U_2^d(t) = \begin{bmatrix} u_{2,0}^d \\ u_{2,1}^d \\ \vdots \\ u_{2,N-1}^d \end{bmatrix} \qquad U_1^s(t) = \begin{bmatrix} u_{1,1}^d \\ u_{1,2}^d \\ \vdots \\ u_{1,N-1}^d \\ K_1 x_{1,N} \end{bmatrix}, \; U_2^s(t) = \begin{bmatrix} u_{2,1}^d \\ u_{2,2}^d \\ \vdots \\ u_{2,N-1}^d \\ K_2 x_{2,N} \end{bmatrix}$$

  - Each agent calculates its optimal control action assuming the other actuates according to the last agreed trajectory U^s

$$\begin{aligned} U_i^* = \min_{U_i} \quad & J_i(U_i, U_{nei}^s, x_i) \\ & x_i(k+1) = A_i x_i(k) + B_{ii} u_i(k) + B_{i,nei} u_{nei}(k) \\ & x_{i,0} = x_i \\ & x_{i,k} \in X_i \\ & x_{i,N} \in \Omega_i \\ & u_{i,k} \in U_i \end{aligned}$$

DMPC scheme for two agents

- Algorithm
  - Then it calculates the wished action for the neighbor, assuming agent *i* will play the action calculated before

$$
\begin{aligned}
U_{nei}^{w} = \min_{U_{nei}} \quad & J_i(U_i^*, U_{nei}, x_i) \\
& x_i(k+1) = A_i x_i(k) + B_{ii} u_i(k) + B_{i,nei} u_{nei}(k) \\
& x_{i,0} = x_i \\
& x_{i,k} \in X_i \\
& x_{i,N} \in \Omega_i \\
& u_{nei,k} \in U_{nei}
\end{aligned}
$$

Each agent has computed two different trajectories using their local model and measurements

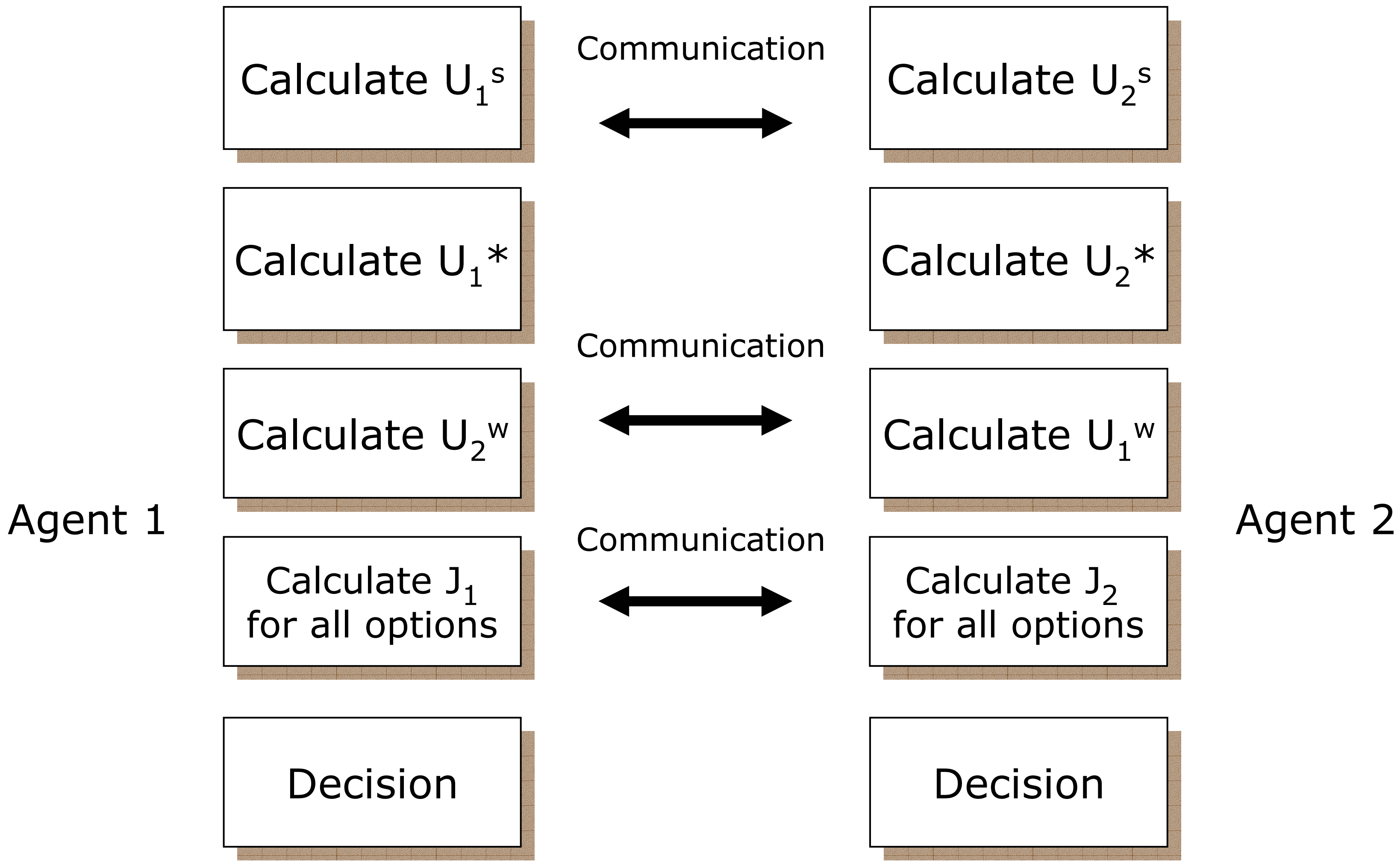
DMPC scheme for two agents

- Algorithm
  - Agents communicate again and build a cooperative game corresponding to the following team problem

A1/ A2	U_2^s	U_2^*	U_2^w
U_1^s Stable option	$J_1\left(x_1(t),U_1^s(t),U_2^s(t)\right)+J_2\left(x_2(t),U_1^s(t),U_2^s(t)\right)$	$J_1\left(x_1(t),U_1^s(t),U_2^*(t)\right)+J_2\left(x_2(t),U_1^s(t),U_2^*(t)\right)$	$J_1\left(x_1(t),U_1^s(t),U_2^w(t)\right)+J_2\left(x_2(t),U_1^s(t),U_2^w(t)\right)$
U_1^* Selfish option	$J_1\left(x_1(t),U_1^*(t),U_2^s(t)\right)+J_2\left(x_2(t),U_1^*(t),U_2^s(t)\right)$	$J_1\left(x_1(t),U_1^*(t),U_2^*(t)\right)+J_2\left(x_2(t),U_1^*(t),U_2^*(t)\right)$	$J_1\left(x_1(t),U_1^*(t),U_2^w(t)\right)+J_2\left(x_2(t),U_1^*(t),U_2^w(t)\right)$
U_1^w Altruist option	$J_1\left(x_1(t),U_1^w(t),U_2^s(t)\right)+J_2\left(x_2(t),U_1^w(t),U_2^s(t)\right)$	$J_1\left(x_1(t),U_1^w(t),U_2^*(t)\right)+J_2\left(x_2(t),U_1^w(t),U_2^*(t)\right)$	$J_1\left(x_1(t),U_1^w(t),U_2^w(t)\right)+J_2\left(x_2(t),U_1^w(t),U_2^w(t)\right)$

  - Agents implement the first global minimum they find
  - The algorithm is repeated the next sampling time

DMPC scheme for two agents



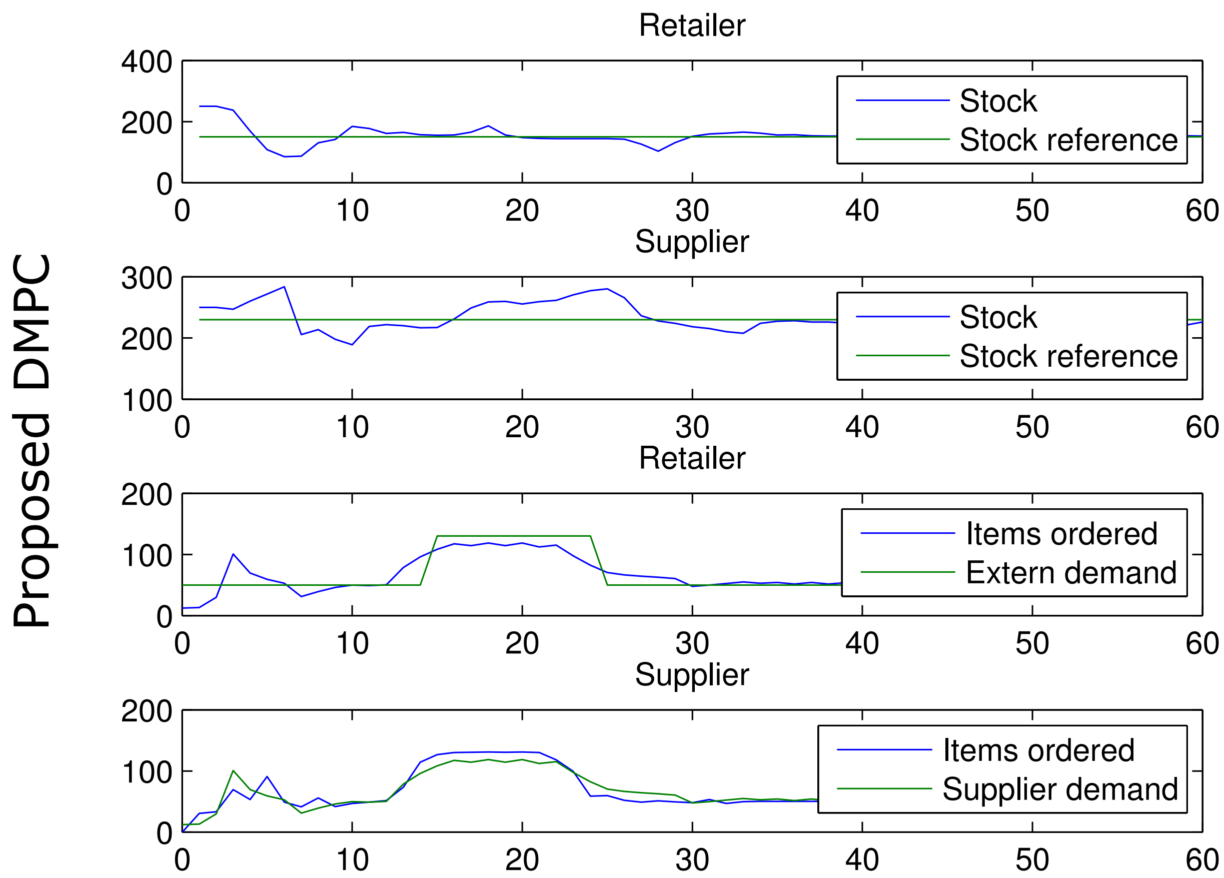
DMPC scheme for two agents

- Application to a supply chain (MIT beer problem)
  - States
    - Stock
    - Unfulfilled order of stock
    - Backlog of unfulfilled orders
  - Manipulated variable
    - Orders
- Simulation scenarios
  - 4 different scenarios
- Comparison
  - Centralized MPC
  - Iterative MPC (based on information broadcast)

DMPC scheme for two agents

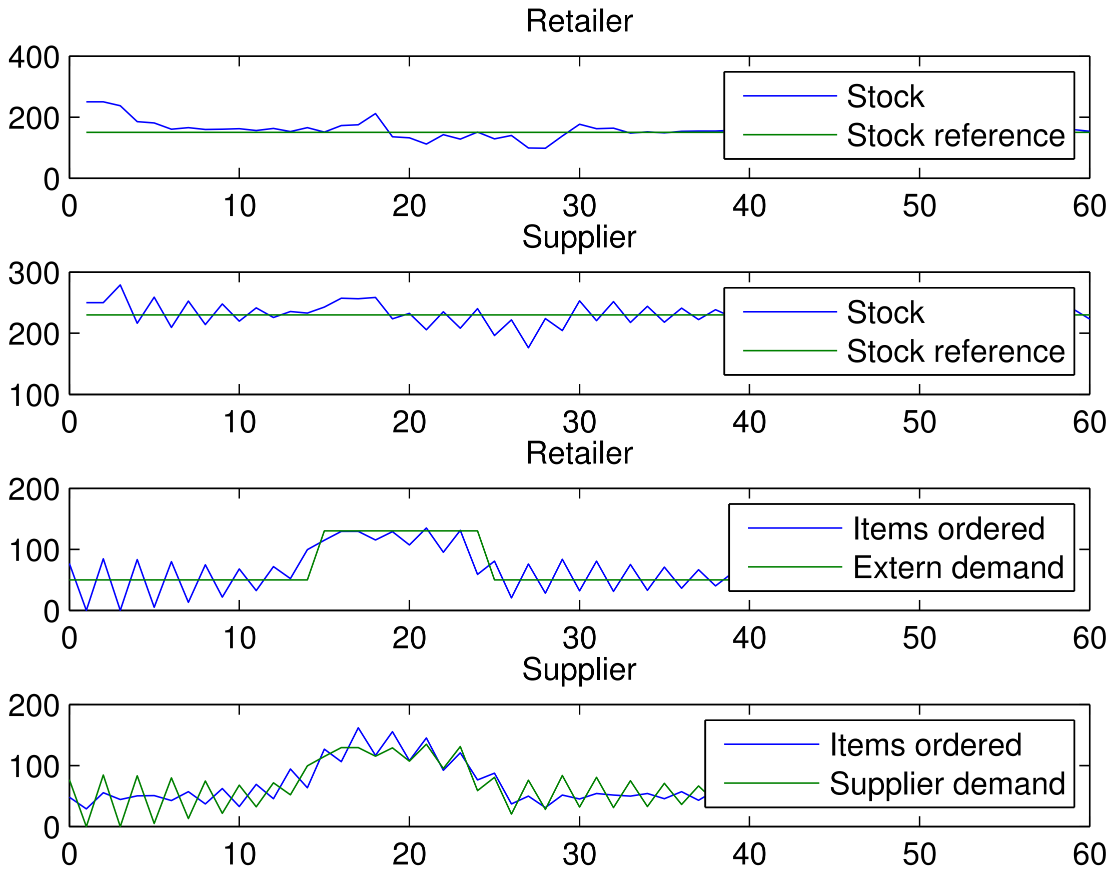
	J	T_{sim}
Centralized	3.6179e+006	1.4187
DMPC	4.9827e+006	0.6246
Iter1	2.1866e+007	0.379715
Iter2	5.6999e+006	0.488593
Iter5	5.8449e+006	1.2611
Iter10	4.1679e+006	1.3750

DMPC scheme for two agents



DMPC scheme for two agents

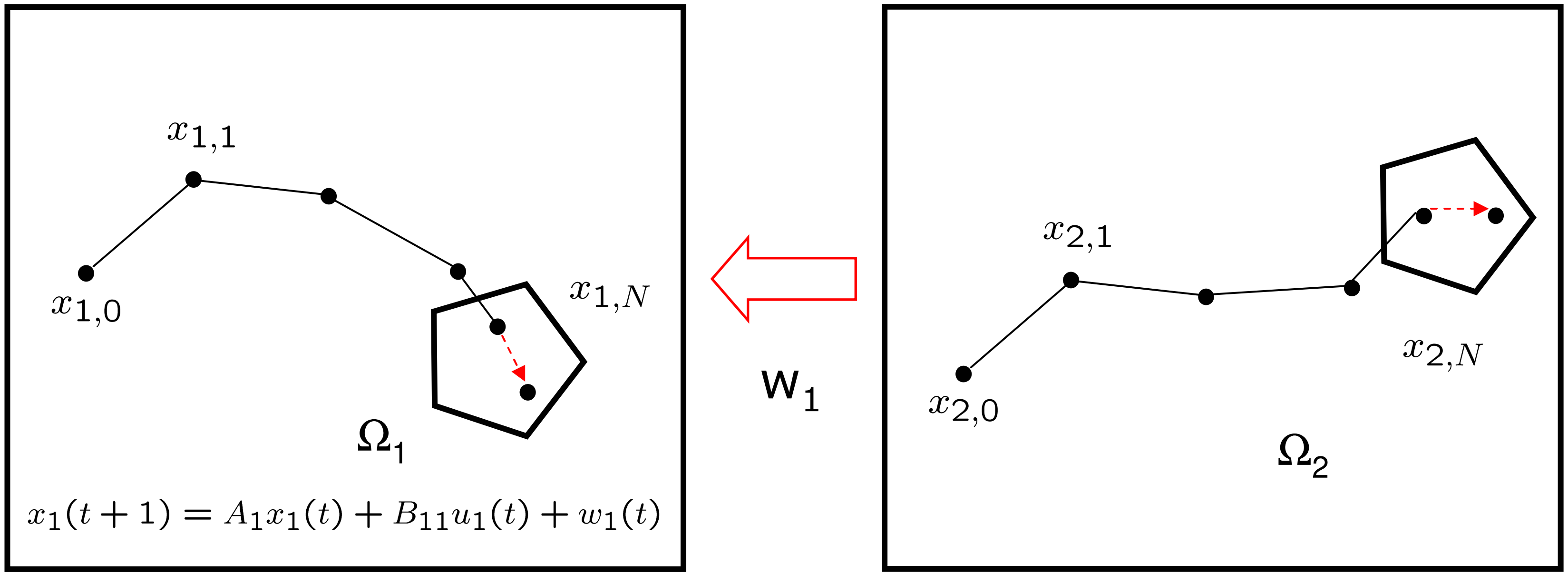
Iter 2



DMPC scheme for two agents

$$U_i^* = \min_{U_i} \; J_i(U_i, U^s_{nei}, x_i)$$
$$x_i(k+1) = A_i x_i(k) + B_{ii} u_i(k) + B_{i,nei} u_{nei}(k)$$
$$x_{i,0} = x_i$$
$$x_{i,k} \in X_i$$
$$x_{i,N} \in \Omega_i$$
$$u_{i,k} \in U_i$$

Terminal region/constraint approach
Robust design approach
Decentralized properties (subspace x_i)



DMPC scheme for two agents

- Stability theorem
  - Terminal cost / Local controller $J_1(x_1, U_1, U_2) = \sum_{k=0}^{N-1} L_1(x_{1,k}, u_{1,k}) + F_1(x_{1,N})$
    - Conditions for each subsystem and also for the overall system

$$F_1((A_1 + B_{11}K_1)x_1 + B_{12}K_2x_2) - F_1(x_1) + L_1(x_1, K_1x_1) \leq 0,\ \forall x_2 \in \Omega_2$$

$$F_2((A_2 + B_{22}K_2)x_2 + B_{21}K_1x_1) - F_1(x_2) + L_1(x_2, K_2x_2) \leq 0,\ \forall x_1 \in \Omega_1$$

$$F((A + BK)x) - F(x) + L(x, Kx) \leq 0,\ \forall x \in \Omega = \Omega_1 \times \Omega_1$$

  - Recursive feasibility

$$\begin{aligned}
&x_1 \in \Omega_1 \rightarrow (A_1 + B_{11}K_1)x_1 + B_{12}K_2x_2 \in \Omega_1,\ \forall x_2 \in \Omega_2 \\
&x_2 \in \Omega_2 \rightarrow (A_2 + B_{22}K_2)x_2 + B_{21}K_1x_1 \in \Omega_2,\ \forall x_1 \in \Omega_1 \\
&K_1x_1 \in U_1,\ \forall x_1 \in \Omega_1 \\
&K_2x_2 \in U_2,\ \forall x_2 \in \Omega_2 \\
&\Omega_1 \in X_1 \\
&\Omega_2 \in X_2
\end{aligned}$$

Note: Local controllers only depend on local state measurements

DMPC scheme for two agents

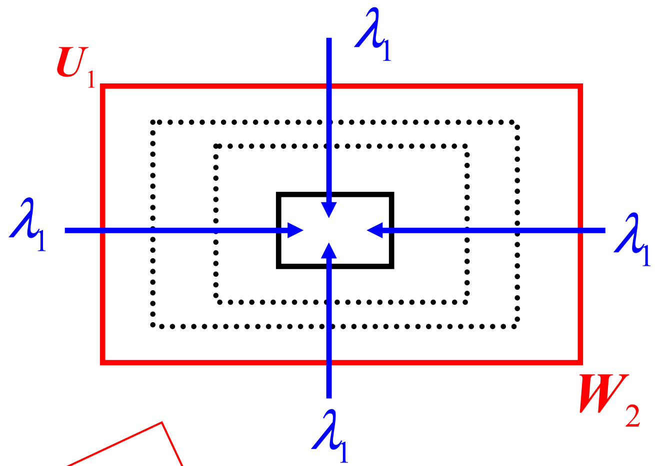
Design procedure based on robust positive invariance

$$\Omega(A, B, D, K, \mathcal{X}, \mathcal{U}, \mathcal{W})$$

$$x \in \Omega \to (A + BK)x + Dw \in \Omega, \ \forall w \in \mathcal{W}$$

$$Kx \in \mathcal{U}$$

$$\Omega \in \mathcal{X}$$



$$\Omega_1(\lambda_1, \lambda_2) = \Omega(A_1, B_{11}, B_{12}, X_1, K_1, \lambda_1 U_1, \lambda_2 U_2)$$
$$\Omega_2(\lambda_1, \lambda_2) = \Omega(A_2, B_{22}, B_{21}, X_2, K_2, \lambda_2 U_2, \lambda_1 U_1)$$

Convex optimization problem

$$\max_{\lambda_1 \in (0,1], \lambda_2 \in (0,1]} f(\Omega_1(\lambda_1, \lambda_2) \times \Omega_2(\lambda_1, \lambda_2))$$

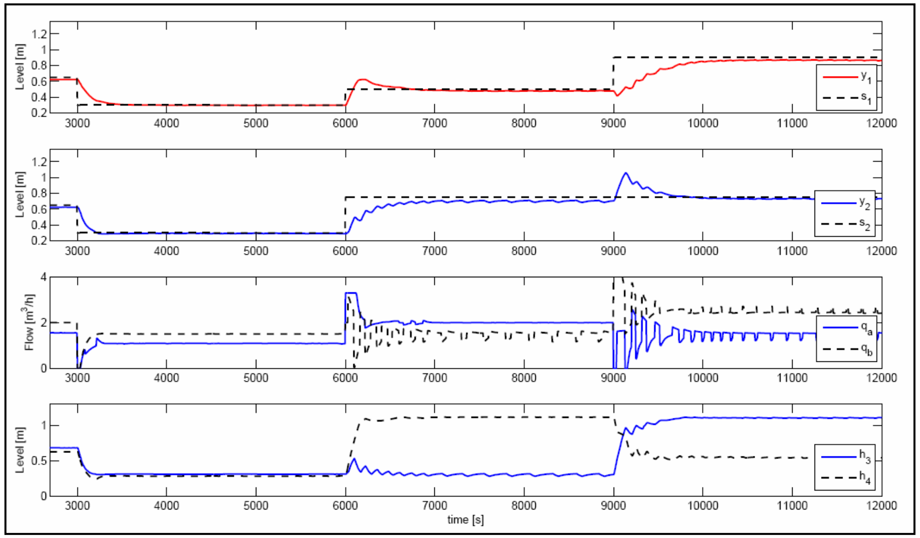
DMPC scheme for two agents

- Local state and model knowledge
- Cooperative solution based on a strategic team problem
- Two/three communications steps
  - Input trajectories
  - Cost function values
- In order to design a stabilizing controller the centralized model is needed
  - And an initial feasible solution!
- Approximate design procedure of jointly invariant sets
  - Parameterization of the input constraints

DMPC scheme for two agents

- HD-MPC four-tank benchmark

"A comparative analysis of distributed MPC techniques applied to the HD-MPC four-tank benchmark". I. Alvarado, D. Limon, D. Muñoz de la Peña, J.M. Maestre, M.A. Ridao, H. Scheu, W. Marquardt, R.R Negenborn, B. De Schutter, F. Valencia and J. Espinosa. Journal of Process Control, 21:5, June 2011, 800-815, Special Issue on Hierarchical and Distributed Model Predictive Control.



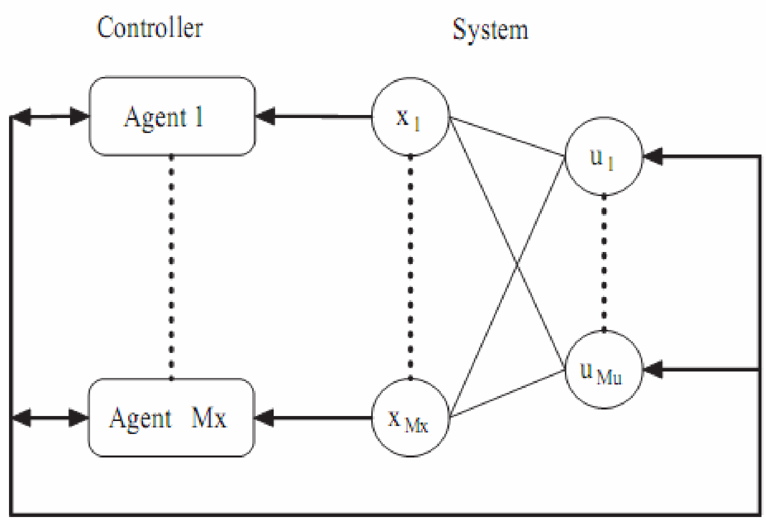
DMPC scheme for multiple agents

- Assumptions
  - There is no coupling between the states of the agents, only in the actuation
  - Each agent has local information about the state and knows how it is affected by the different inputs
  - Input and state constraints
  - Inputs are not assigned to agents

$$x_i(t+1) = A_i x_i(t) + \sum_{j \in n_i} B_{ij} u_j(t)$$

$$x_i \in \mathcal{X}_i, \; i = 1, \ldots, M_x$$

$$u_j \in \mathcal{U}_j, \; j = 1, \ldots, M_u$$



DMPC scheme for multiple agents

Agents optimize according to a local cost function

$$J_i(x_i, \{U_j\}_{j\in n_i}) = \sum_{k=0}^{N-1} L_i(x_{i,k}, \{u_{j,k}\}_{j\in n_i}) + F_i(x_{i,N})$$

$$L_i(x_i, \{u_j\}_{j\in n_i}) = x_i^T Q_i x_i + \sum_{j\in n_i} u_j^T R_{ij} u_j$$

$$F_i(x_i) = x_i^T P_i x_i$$

Control objective

$$\sum_{i=1}^{M_x} J_i(x_i(t), \{U_j(t)\}_{j\in n_i})$$

GLOBAL PERFORMANCE INDEX

The different agents must reach an agreement on the value of the shared inputs

DMPC scheme for multiple agents

- Proposed DMPC scheme
  - Subsystems coupled through the inputs
  - Each agent has only partial information of the system
  - Low communicational requirements
  - Cooperative solution
    - Cooperative algorithm from a game theory point of view
  - Guaranteed closed-loop stability properties

- Direct extension of the previous algorithm is not possible because of the combinatorial explosion
  - N agents with q proposals lead to q^N options!

- Negotiation based scheme

DMPC scheme for multiple agents

- Algorithm
  - Each agent receives its state info x_i
  - The agents communicate (if needed) to evaluate the initial value of the input trajectories U_s (shifted inputs) from the latest decided input

$$U_i^d(t-1) = \begin{bmatrix} u_{j,0}^d \\ u_{j,1}^d \\ \vdots \\ u_{j,N-1}^d \end{bmatrix} \qquad U_j^s(t) = \begin{bmatrix} u_{j,1}^d \\ u_{j,2}^d \\ \vdots \\ u_{j,N-1}^d \\ \sum_{p \in m_j} K_{jp} x_{p,N} \end{bmatrix}$$

  - A number of proposals are made by a set of agents
    - A proposal consists of a future trajectory for a subset of inputs
    - A proposal is accepted if and only if it improves the costs for all the agents affected by that control action
    - After a predefined number of proposals are made, the latest agreed input trajectory is applied

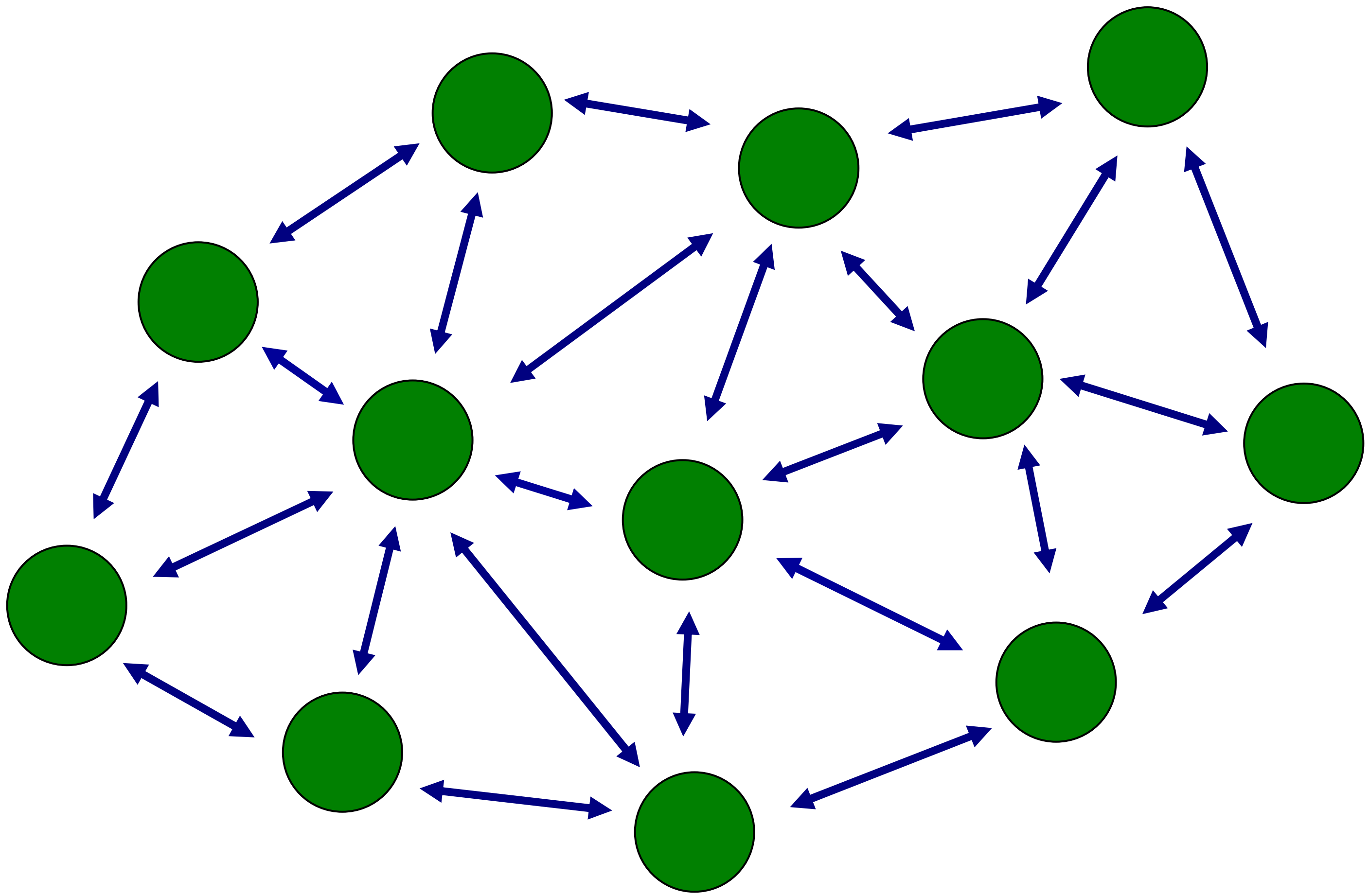
DMPC scheme for multiple agents

- Algorithm
  - In order to make a proposal, each agent calculates the optimal control action for a (sub)set of inputs that affect its dynamics

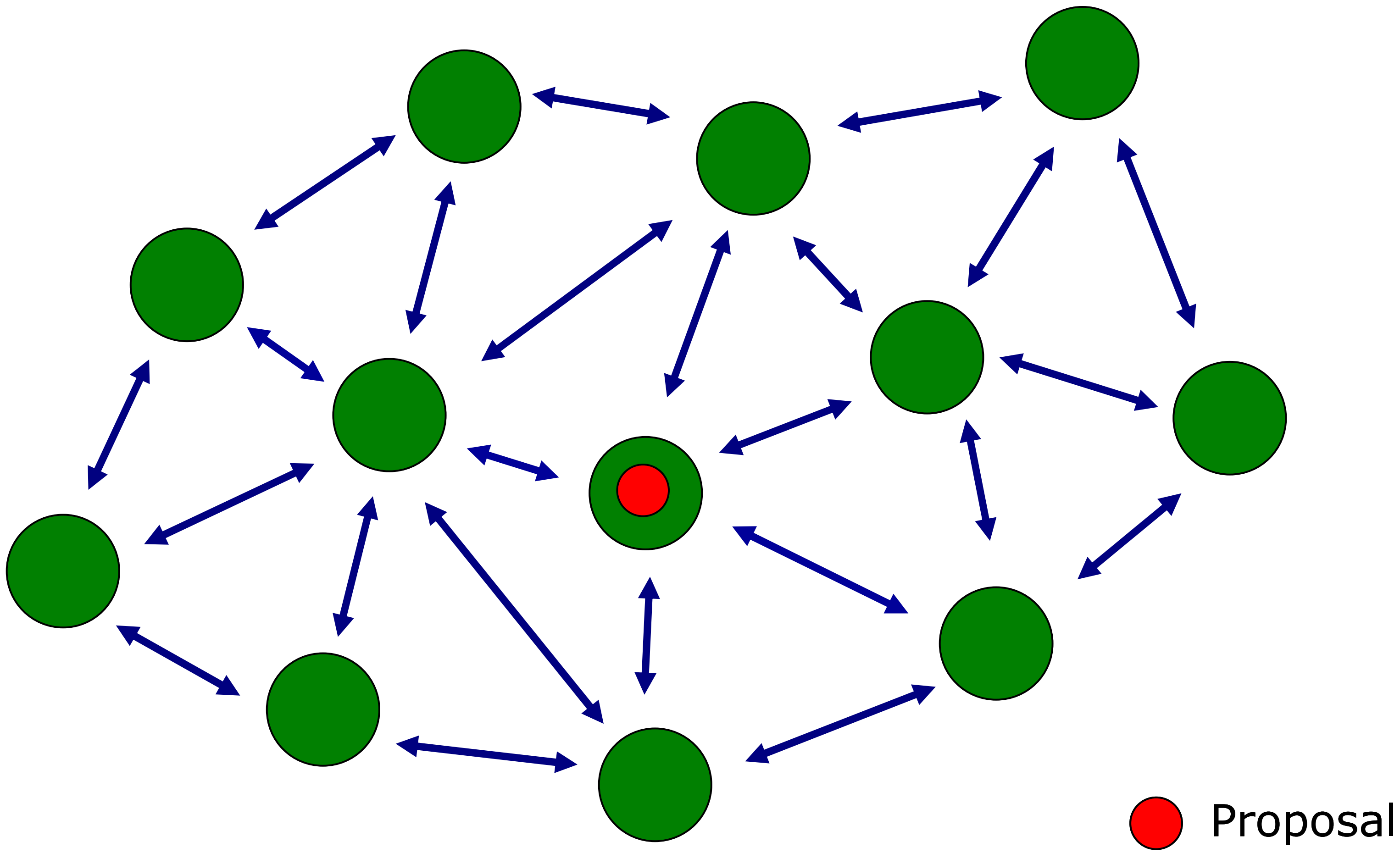
$$
\begin{aligned}
\{U_j^p(t)\}_{j\in n_p} = \; & \arg \min_{\{U_j\}_{j\in n_p}} J_p(x_p, \{U_j\}_{j\in n_p}) \\
& s.t. \\
& x_{p,k+1} = A_p x_{p,k} + \textstyle\sum_{j\in n_p} B_{pj} u_{j,k} \\
& x_{p,0} = x_i(t) \\
& x_{p,k} \in \mathcal{X}_p, \;\; k = 0, \ldots N \\
& u_{j,k} \in \mathcal{U}_j, \;\; k = 0, \ldots N-1, \;\; \forall j \in n_p \\
& x_{p,N} \in \Omega_p \\
& U_j = U_j^s(t), \;\; \forall j \notin P_p
\end{aligned}
$$

Different communication protocols: Round robin, asynchronous...

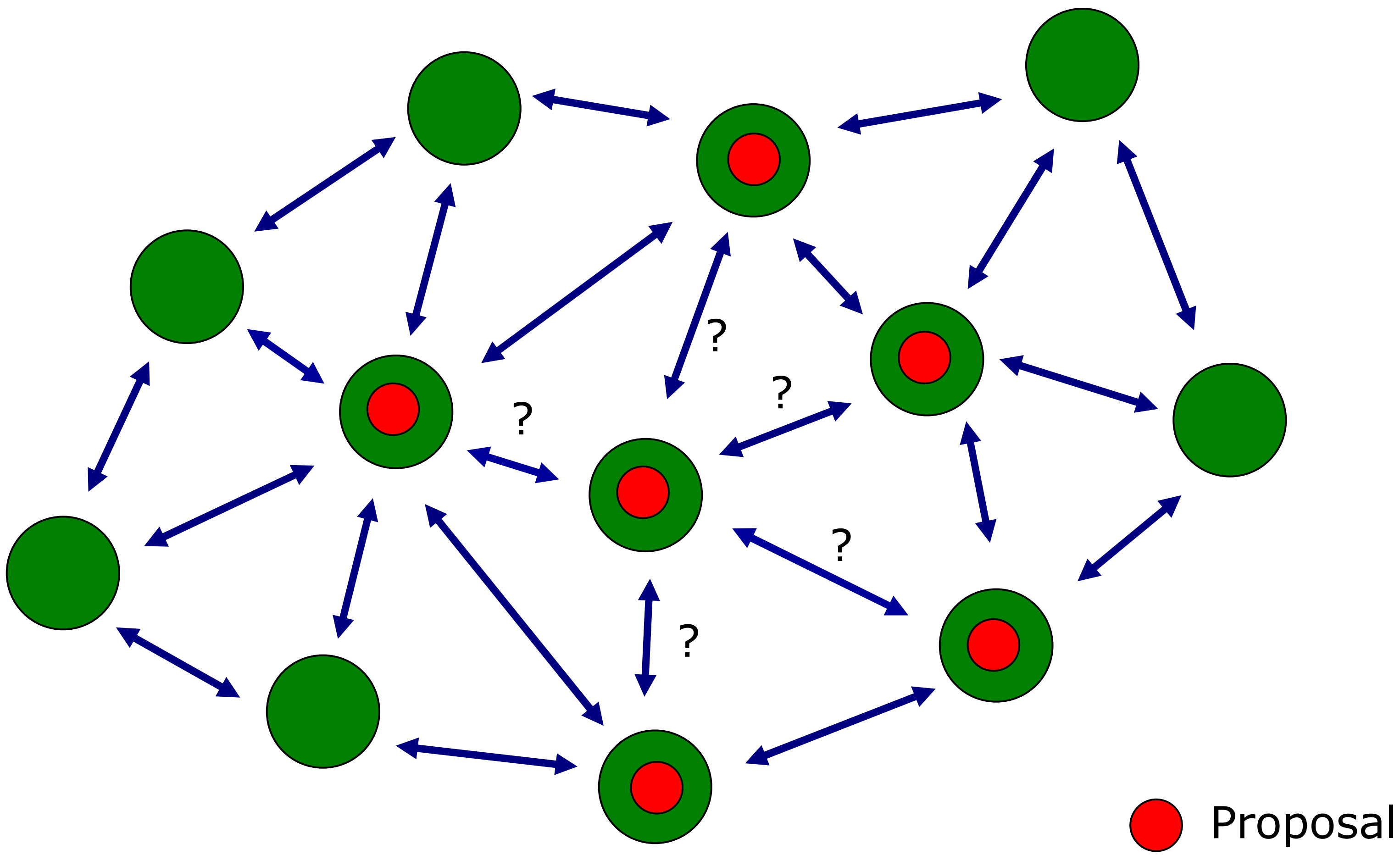
DMPC scheme for multiple agents



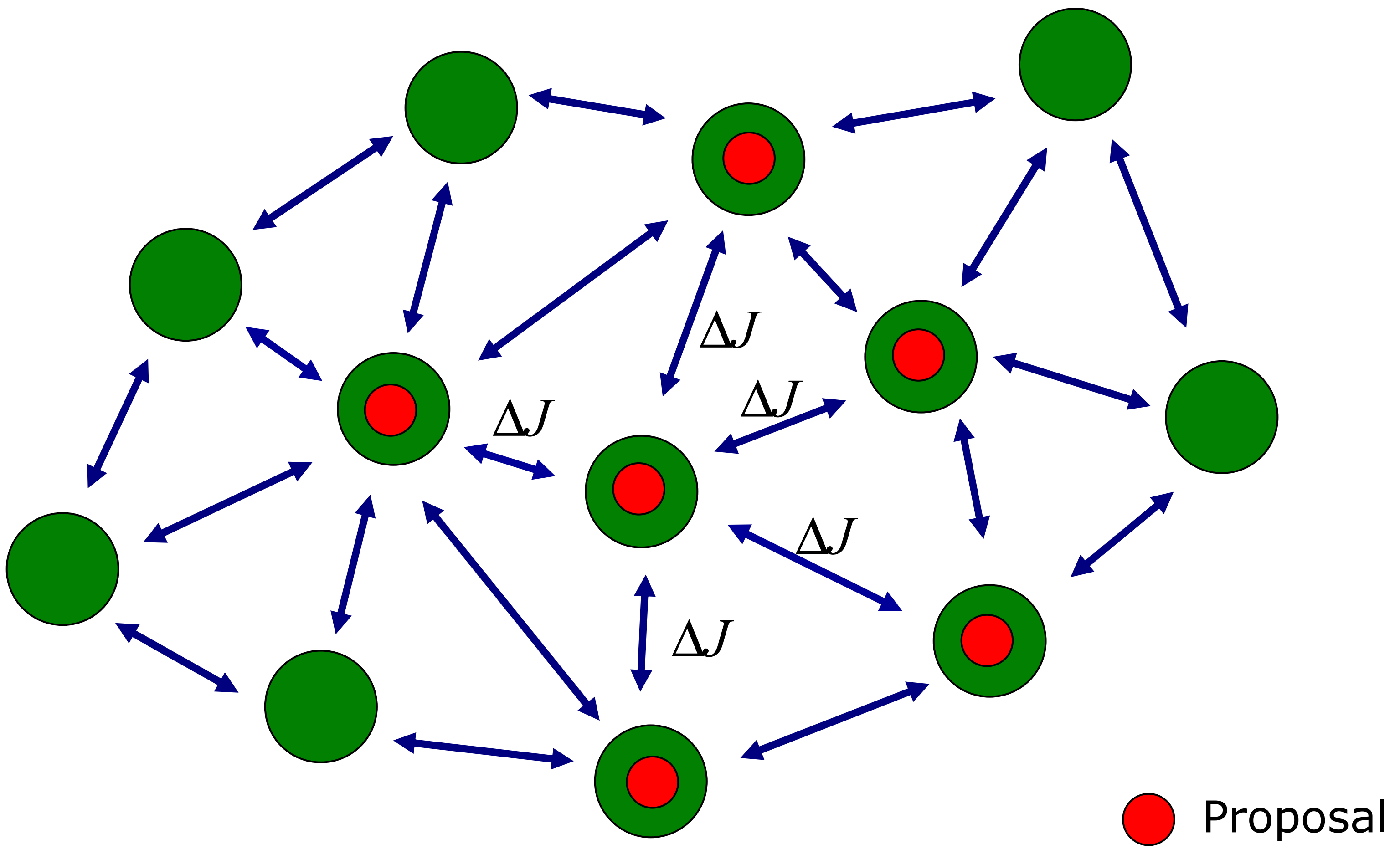
DMPC scheme for multiple agents



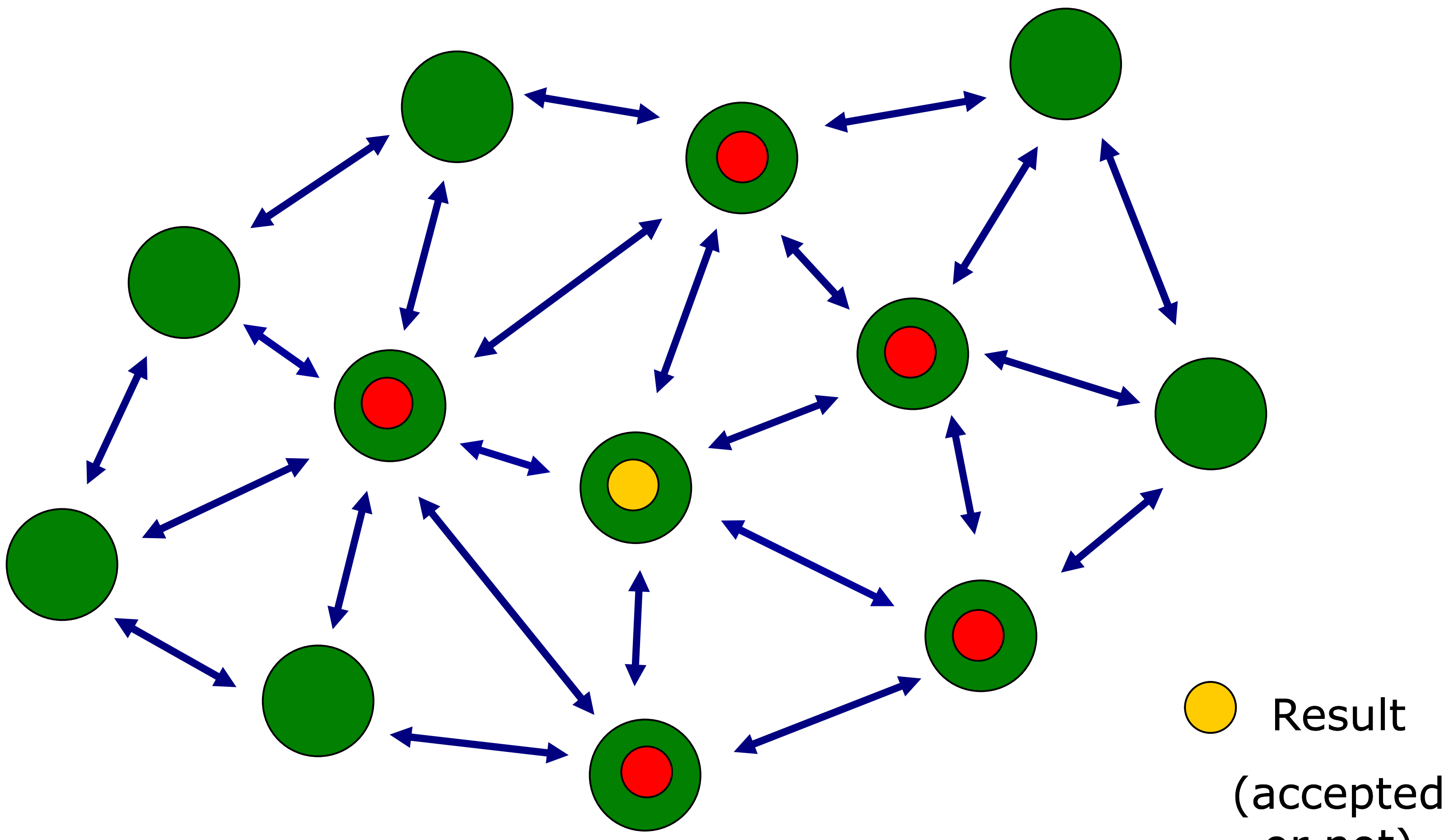
DMPC scheme for multiple agents



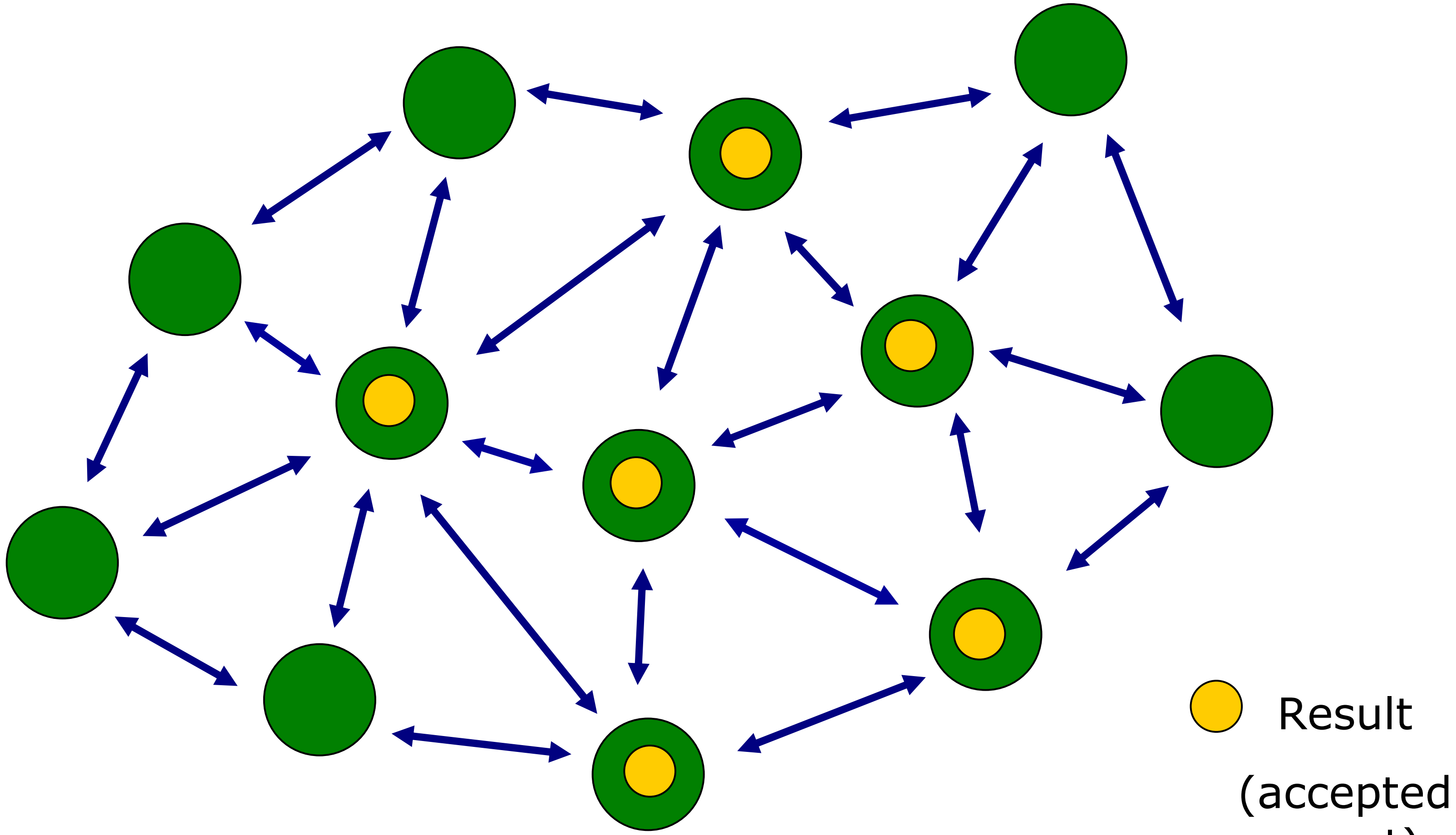
DMPC scheme for multiple agents



DMPC scheme for multiple agents

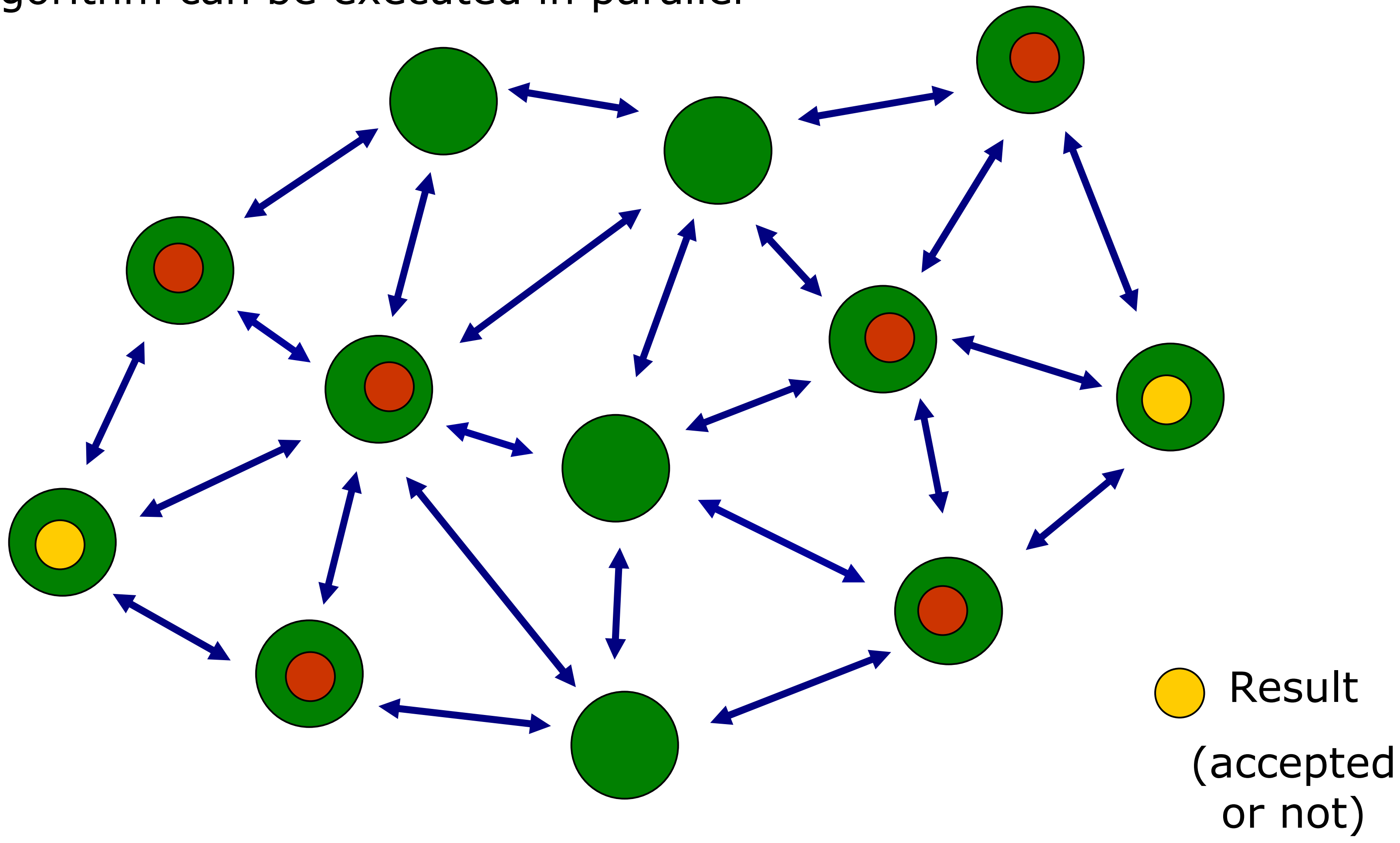


DMPC scheme for multiple agents



DMPC scheme for multiple agents

Algorithm can be executed in parallel



DMPC scheme for multiple agents

- Stability theorem
  - Terminal cost / Local controllers
    - Stabilizing linear controller (centralized or decentralized)

$$\sum_{i=1}^{M_x} F_i(A_i x_i + \sum_{j\in n_i} B_{ij} \sum_{p\in m_j} K_{jp} x_p) - F_i(x_i) + L_i(x_i, \{\sum_{p\in m_j} K_{jp} x_p\}_{j\in n_i}) \leq 0$$

  - Recursive feasibility

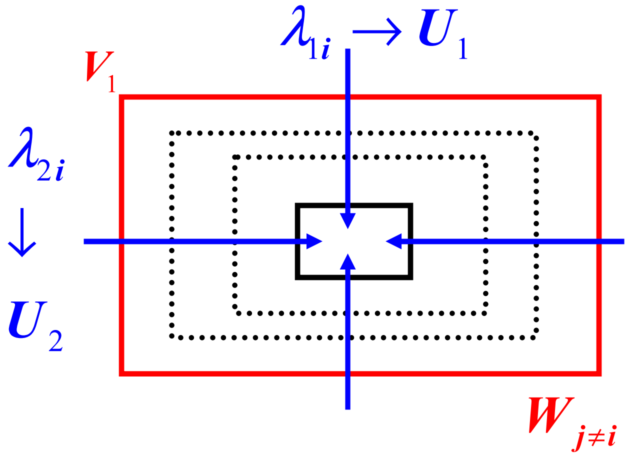
If $x_i \in \Omega_i$ for all i then

$$A_i x_i + \sum_{j\in n_i} B_{ij} \sum_{p\in m_j} K_{jp} x_p \in \Omega_i$$
$$\sum_{p\in m_j} K_{jp} x_p \in U_j$$
$$\Omega_i \in X_i$$

  - Jointly invariant set: robust stability w.r.t. neighbors
    - Standard LMI design techniques
    - Centralized model is needed

DMPC scheme for multiple agents

$$x_i(t+1) = A_i x_i(t) + B_i v_i(t) + E_i w_i(t)$$



$$B_i v_i(t) = \sum_{j \in n_i} B_{ij} K_{ji} x_i$$

$$K_{ji} x_i \in \lambda_{ji} \mathcal{U}_j$$

$$\sum_{i \in m_j} \lambda_{ji} \leq 1$$

$$E_i w_i(t) = \sum_{j \in n_i} B_{ij} \sum_{p \in m_j - \{i\}} K_{jp} x_p$$

$$V_i(\Lambda) = \lambda_{1i}\mathcal{U}_1 \times \lambda_{2i}\mathcal{U}_2 \times \ldots \times \lambda_{M_u i}\mathcal{U}_{M_u}$$
$$W_i(\Lambda) = (\sum_{p \in m_1 - \{i\}} \lambda_{1p})\mathcal{U}_1 \times (\sum_{p \in m_2 - \{i\}} \lambda_{2p})\mathcal{U}_2 \times \ldots \times (\sum_{p \in m_{M_u} - \{i\}} \lambda_{M_u p}\mathcal{U}_{M_u})$$

Convex optimization problem

$$\max_{\lambda_{ji}} f(\Omega_1 \times \Omega_2 .. \times \Omega_{M_x})$$

$$\Omega_i = \Omega(A_i, B_i, E_i, \mathcal{X}_i, K_i, \mathcal{V}_i(\Lambda), \mathcal{W}_i(\Lambda))$$

$$\lambda_{ji} \in (0, 1), \ \forall j, i$$

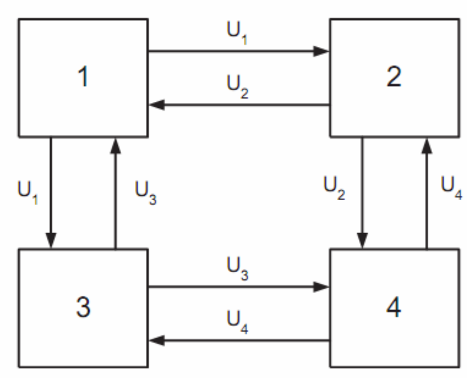
$$\sum_{i \in m_j} \lambda_{ji} \leq 1, \ \forall i$$

DMPC scheme for multiple agents

- Local state and model knowledge
- Cooperative solution based on negotiation
- Multiple communications with neighbors
  - Input trajectories
  - Cost function values
- Parallel implementation
- In order to design a stabilizing controller the centralized model is needed
  - And an initial feasible solution!
- Approximate design procedure of jointly invariant sets
  - Parameterization of the input constraints

DMPC scheme for multiple agents

- Example
  - Four coupled systems



$$A_1 = \begin{bmatrix} 1 & 0.8 \\ 0 & 0.7 \end{bmatrix}, B_{11} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, B_{12} = \begin{bmatrix} 0 \\ 0.15 \end{bmatrix}, B_{13} = \begin{bmatrix} 0 \\ 0.15 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 0.6 \\ 0 & 0.7 \end{bmatrix}, B_{21} = \begin{bmatrix} 0 \\ 0.15 \end{bmatrix}, B_{22} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, B_{24} = \begin{bmatrix} 0 \\ 0.15 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0.9 \\ 0 & 0.8 \end{bmatrix}, B_{31} = \begin{bmatrix} 0 \\ 0.15 \end{bmatrix}, B_{33} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, B_{34} = \begin{bmatrix} 0 \\ 0.15 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 1 & 0.8 \\ 0 & 0.5 \end{bmatrix}, B_{42} = \begin{bmatrix} 0 \\ 0.15 \end{bmatrix}, B_{43} = \begin{bmatrix} 0 \\ 0.15 \end{bmatrix}, B_{44} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

  - Bounds on states and inputs

$$|x_1|_\infty \leq 1,\ |x_2|_\infty \leq 2,\ |x_3|_\infty \leq 1,\ |x_4|_\infty \leq 2$$
$$|u_1|_\infty \leq 1,\ |u_2|_\infty \leq 1,\ |u_3|_\infty \leq 1,\ |u_4|_\infty \leq 1$$

  - Cost functions

$$J_i(x_i, \{U_j\}_{j\in n_i}) = \sum_{k=0}^{N-1} L_i(x_{i,k}, \{u_{j,k}\}_{j\in n_i}) + F_i(x_{i,N})$$

$$L_i(x_i, \{u_j\}_{j\in n_i}) = x_i^T Q_i x_i + \sum_{j\in n_i} u_j^T R_{ij} u_j$$

$$Q_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, R_{ij} = 10$$

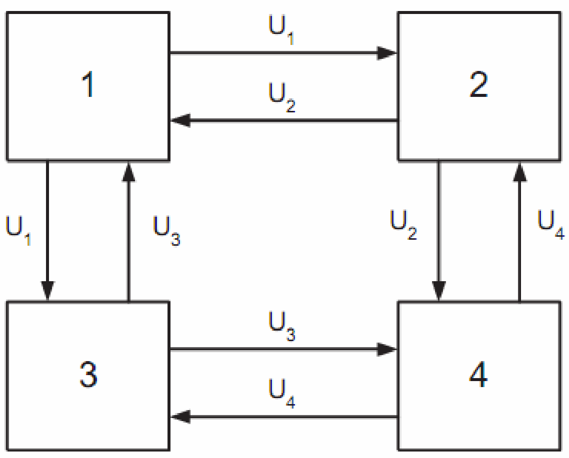
$$F_i(x_i) = x_i^T P_i x_i$$

Ki, Pi have to be designed

DMPC scheme for multiple agents

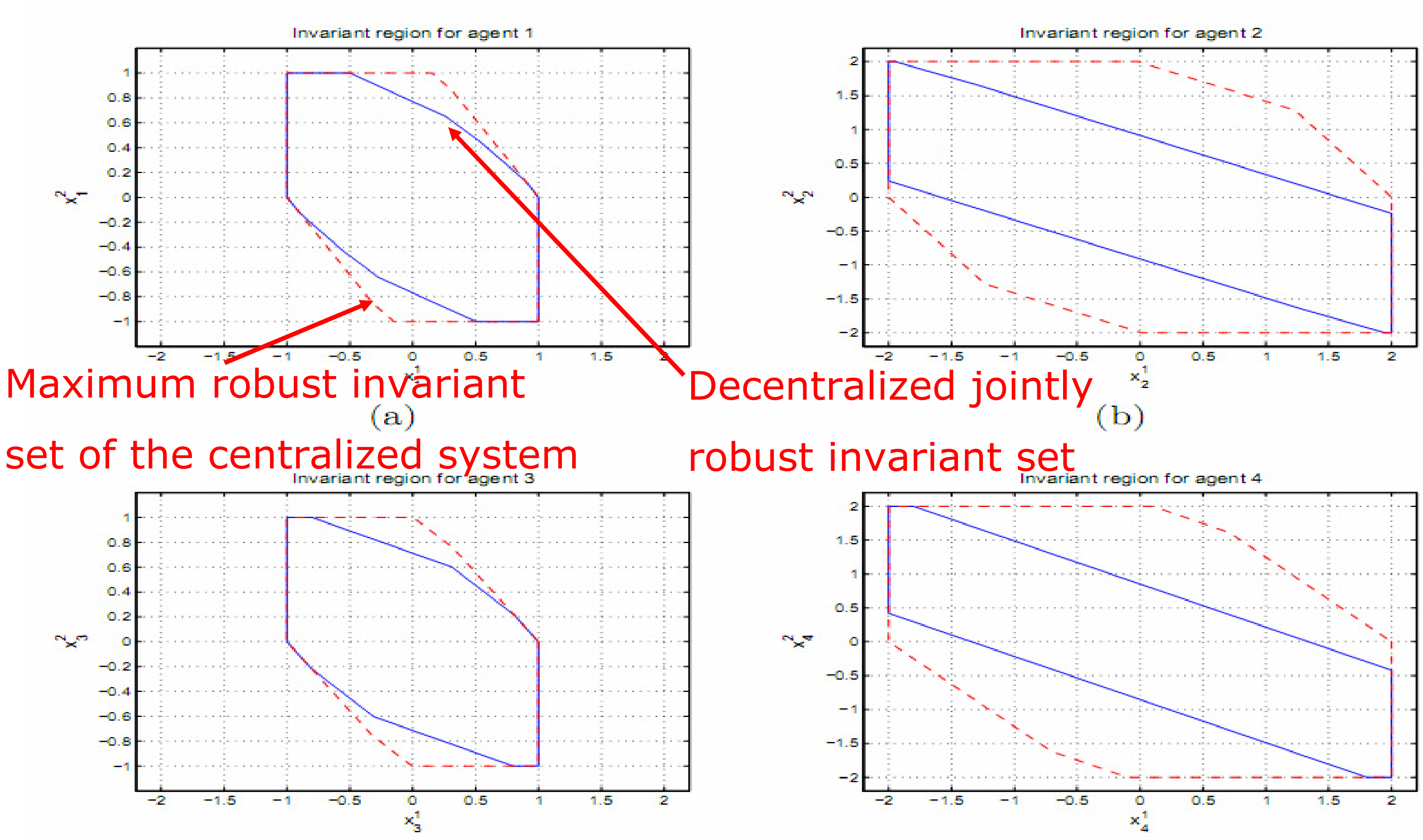
LMI design:

$$K = \begin{bmatrix} -0.2732 & -0.5935 & -0.0065 & -0.0112 & -0.0055 & -0.0151 & 0 & 0 \\ -0.0122 & -0.0263 & -0.2891 & -0.5024 & 0 & 0 & -0.0138 & -0.0216 \\ -0.0025 & -0.0052 & 0 & 0 & -0.2463 & -0.6878 & -0.0027 & -0.0042 \\ 0 & 0 & -0.0118 & -0.0204 & -0.0100 & -0.0276 & -0.3081 & -0.4845 \end{bmatrix}$$

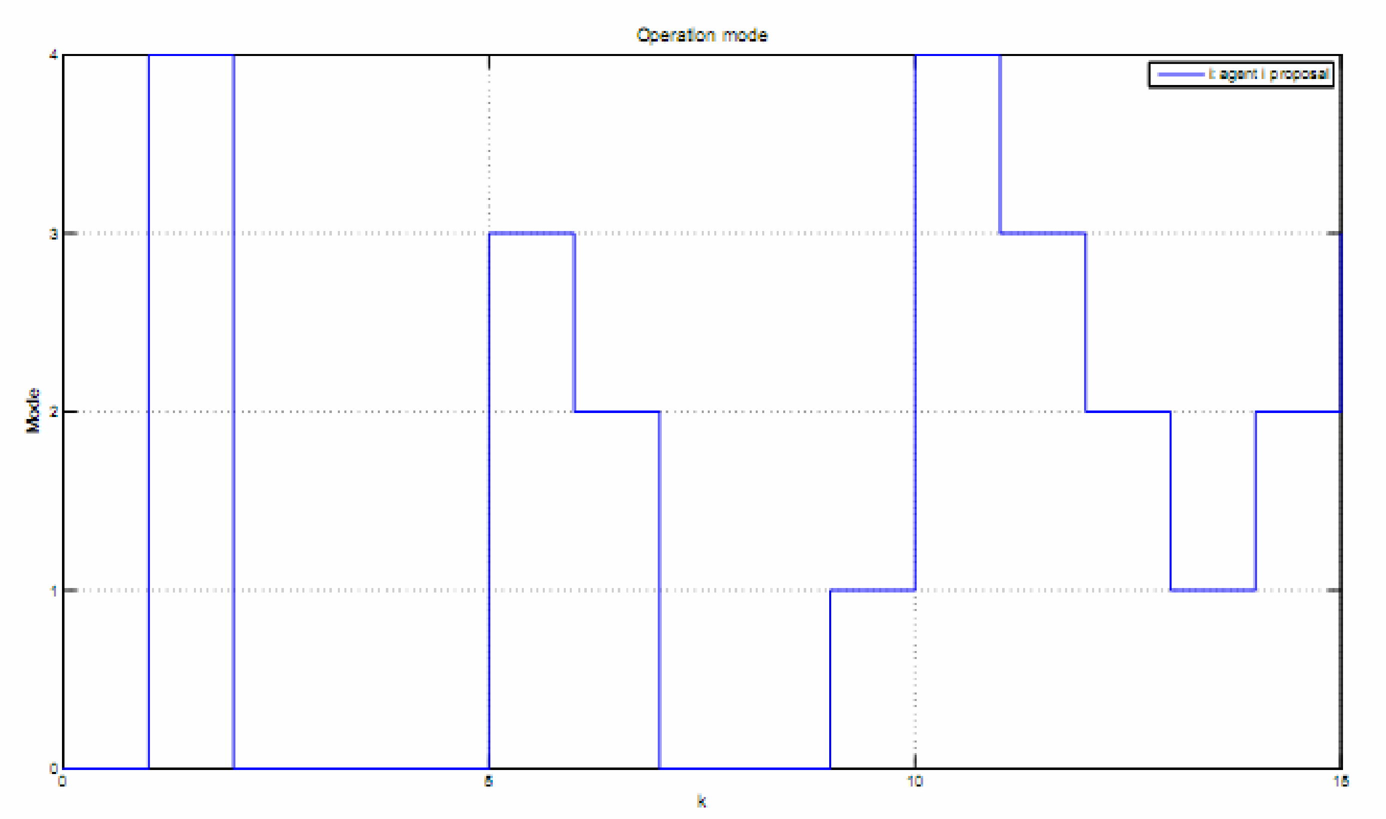


$$P = \begin{bmatrix} 4.9218 & 5.7645 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5.7645 & 11.3025 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5.6585 & 5.4284 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5.4284 & 8.8248 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4.4536 & 5.8125 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5.8125 & 13.7428 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5.6128 & 5.8004 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5.8004 & 8.9564 \end{bmatrix}$$

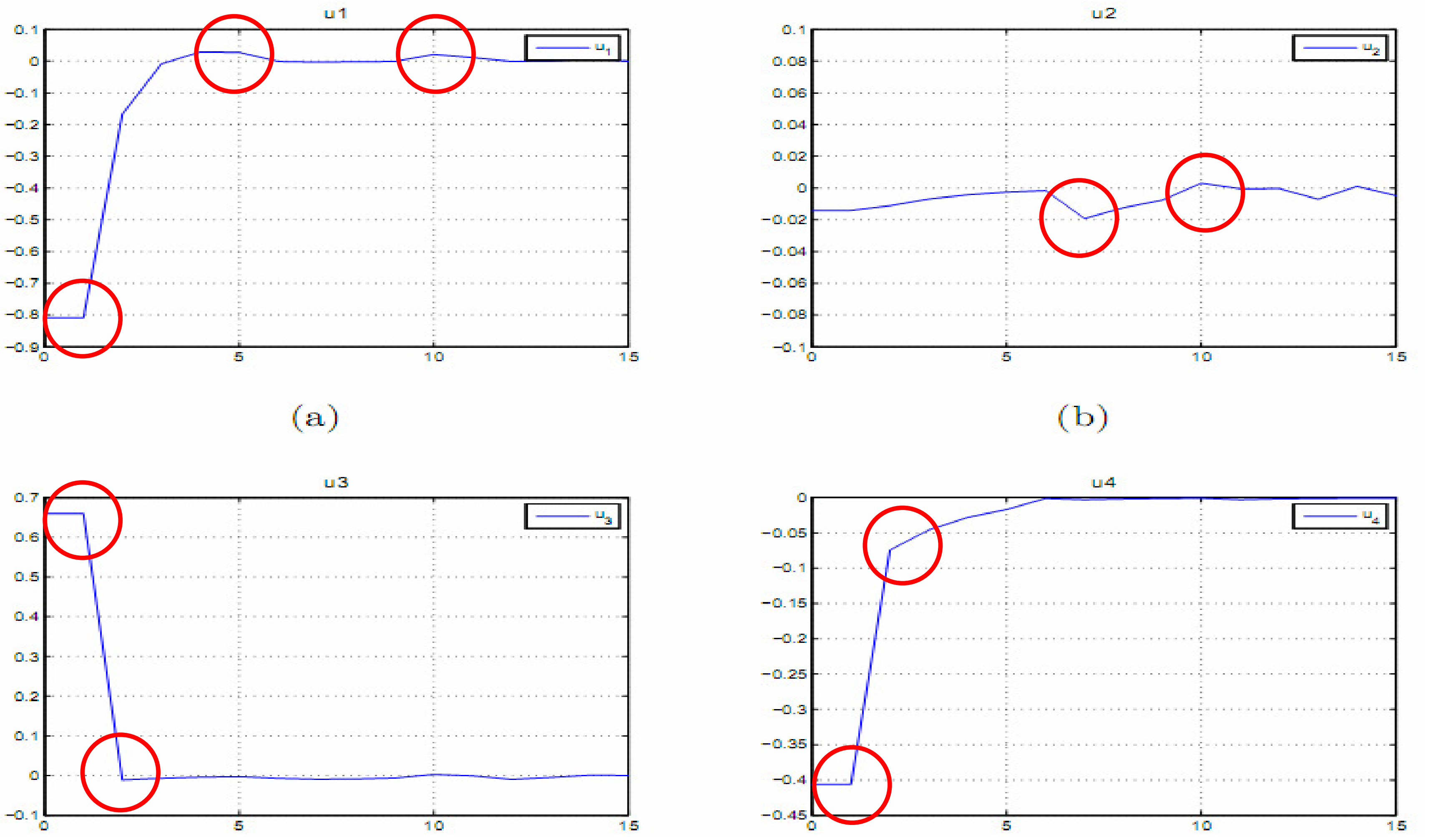
DMPC scheme for multiple agents



DMPC scheme for multiple agents



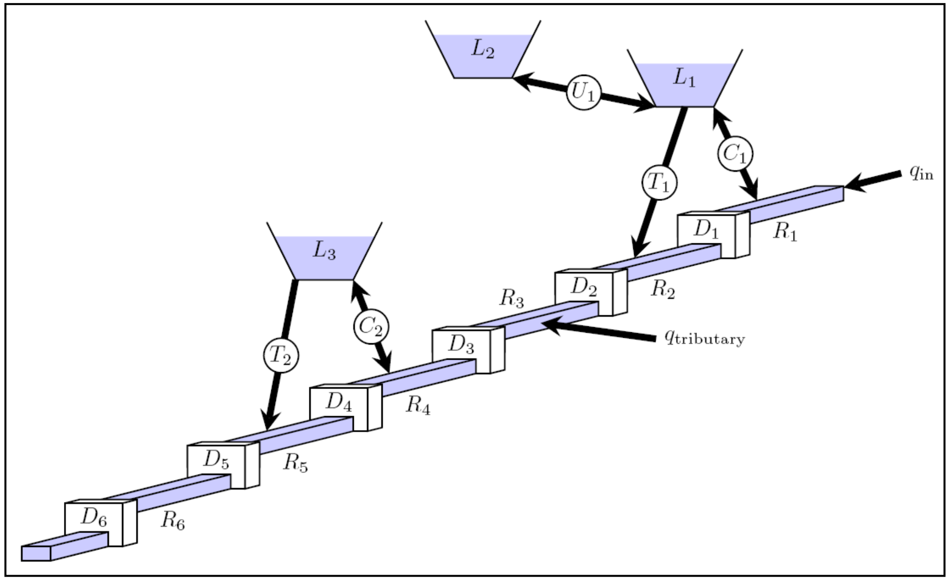
DMPC scheme for multiple agents



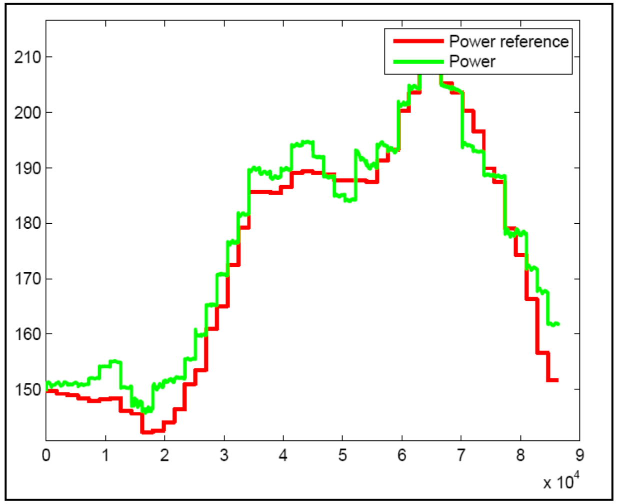
DMPC scheme for multiple agents

- Control benchmark of a Hydro Power Plant

Nonlinear system
Power reference tracking
Profit maximization



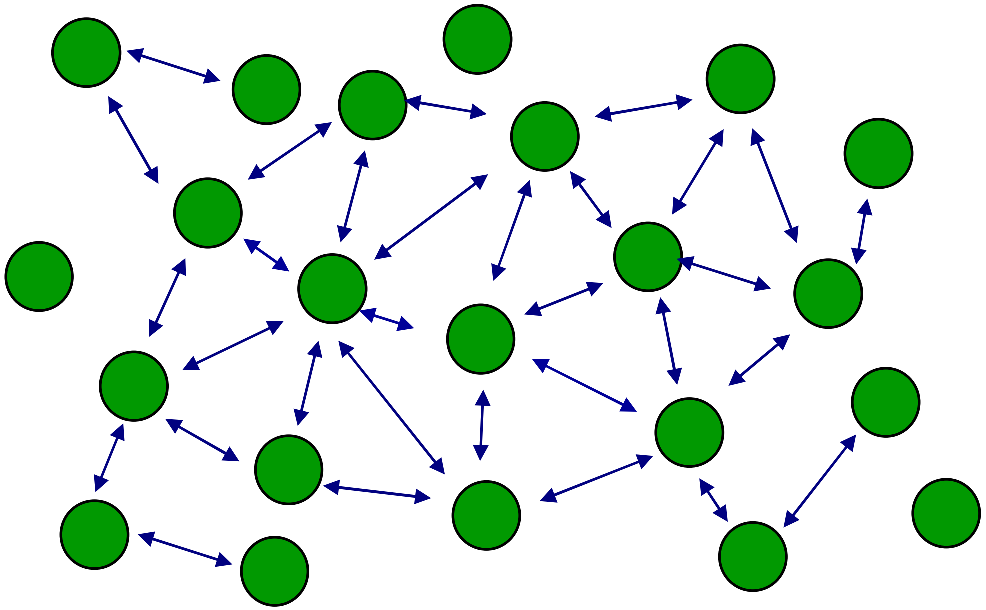
DMPC with 8 agents
Linear model
Two time scales (state decoupling)



Food for thought…

Questions

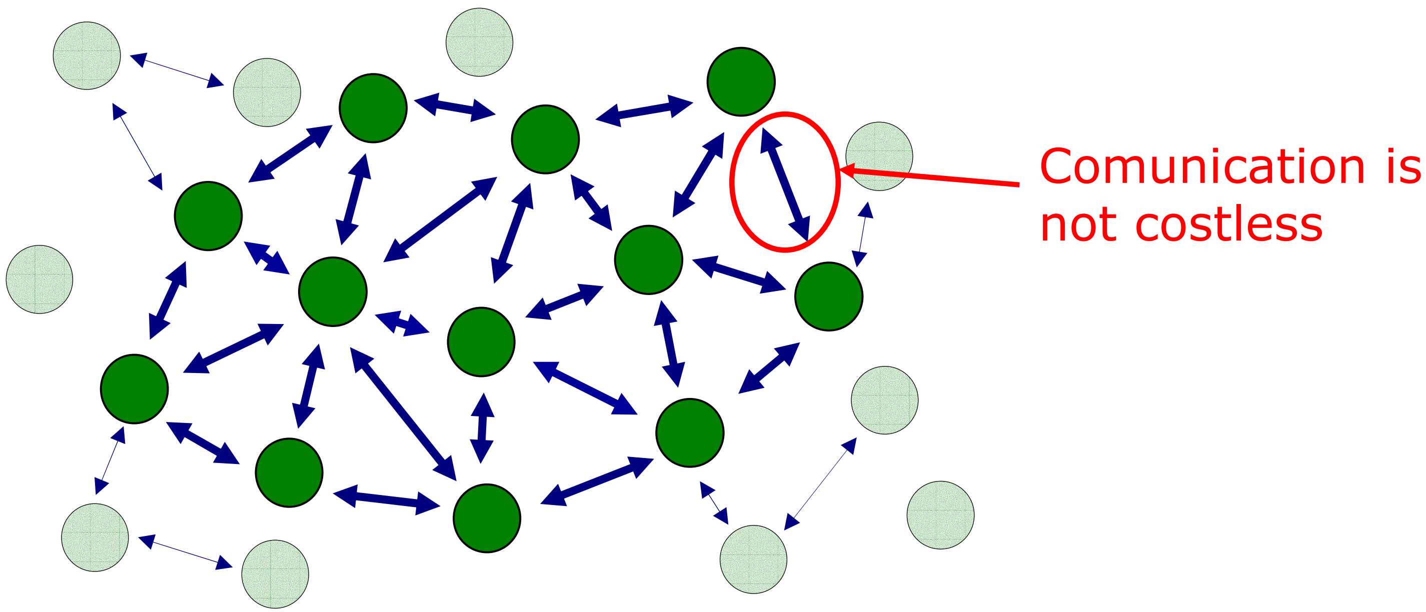
- -Do all the links have to be enabled all the time?
- -How to divide profits/costs between the agents?
- -Which are the most relevant agents/links?



Food for thought…

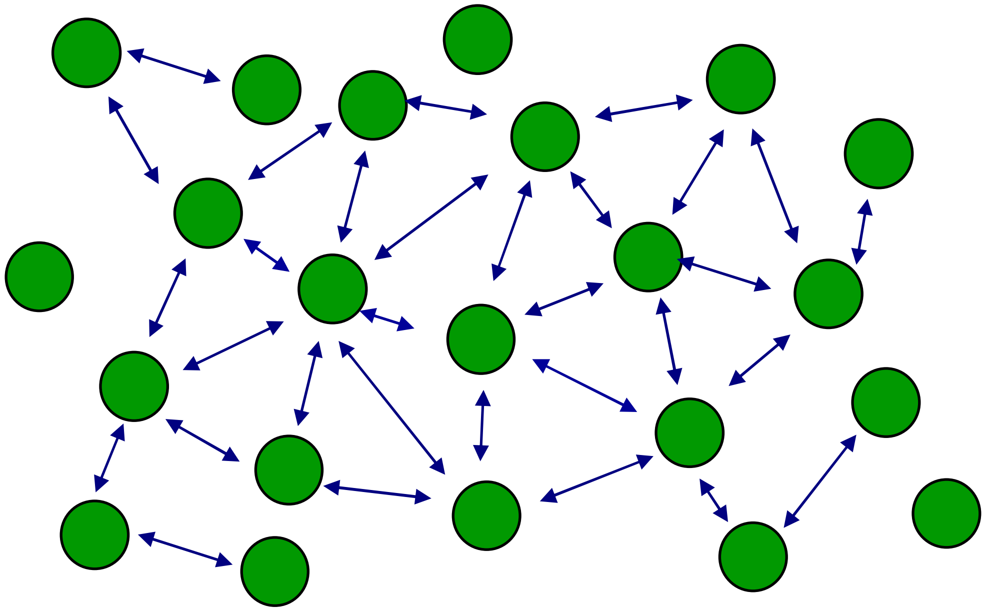
- A cooperative game is defined by…
  - A set of agents *N={1,2,…,n}*
    - *Separated into coalitions S*
  - A *characteristic* function *v* that assigns a value to each of the possible 2^n coalitions
    - *v(S)* represents the cost to reach the common goal without the assistance of the agents out of the coalition

V(s)



Food for thought…

An application of Cooperative Game Theory to Distributed Control. J. M. Maestre, D. Muñoz de la Peña, A. Jiménez Losada, E. Algaba Durán, E. F. Camacho. Proceedings of the 18th IFAC World Congress.



Related publications

- Distributed model predictive control based on a cooperative game. J. M. Maestre, D. Muñoz de la Peña, E. F. Camacho. Optimal Control Applications and Methods, 32:2, March/April 2011, 153–176.

- Distributed model predictive control based on agent negotiation, J.M. Maestrea, D. Muñoz de la Peña, E.F. Camacho and T. Alamo. Journal of Process Control, 21:5, June 2011, 685-697.

- A comparative analysis of distributed MPC techniques applied to the HD-MPC four-tank benchmark. I. Alvarado, D. Limon, D. Muñoz de la Peña, J.M. Maestre, M.A. Ridao, H. Scheu, W. Marquardt, R.R Negenborn, B. De Schutter, F. Valencia and J. Espinosa. Journal of Process Control, 21:5, June 2011, 800-815.

- An application of Cooperative Game Theory to Distributed Control. J. M. Maestre, D. Muñoz de la Peña, A. Jiménez Losada, E. Algaba Durán, E. F. Camacho. Proceedings of the 18th IFAC World Congress.

The end

Thanks for your attention!

Questions, suggestions, comments...