AACHENER VERFAHRENSTECHNIK

Distributed Model Predictive Control by Primal Decomposition

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“Hierarchical and Distributed Model-Predictive Control’

RWTH

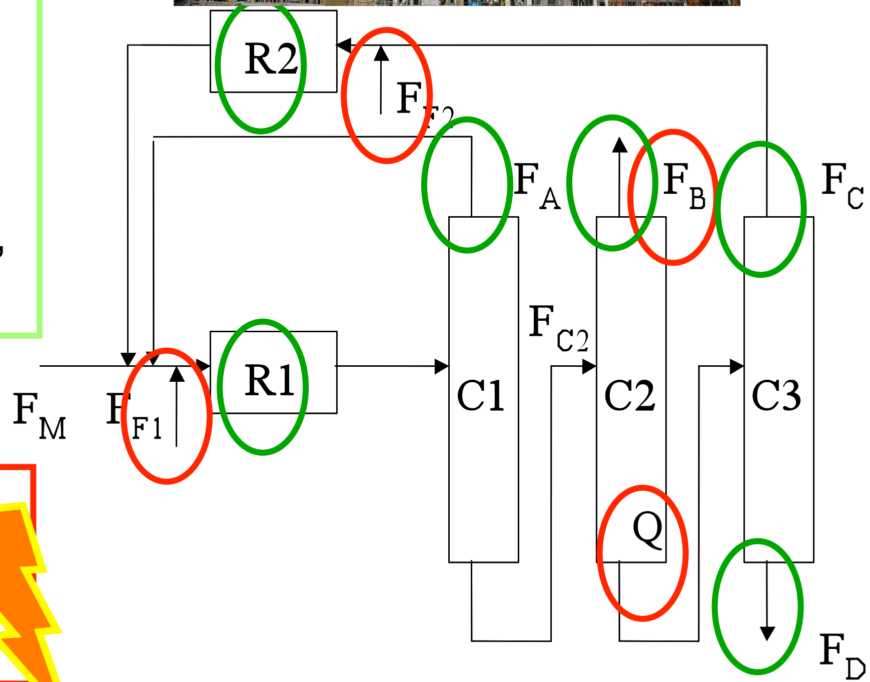
Motivation and Background

- **Chemical & energy process plants**
  - large-scale, structured
  - nonlinear, stiff
- **Process control and operations**
  - industrial state of the art
    - decentralized (PID) control
      & supervisory control
    - linear (centralized) MPC using step response or state space models from plant tests)
  - (selected) research activities
    - nonlinear centralized MPC and RHE using first principles models
    - dynamic real-time optimization (DRTO)
    - hierarchical or/and decentralized optimal control (MPC, DRTO) and matching nonlinear data/model reconciliation & state estimation

Industrial Case Study (1)

Large-scale industrial process (Shell):

- How should decentralized control scheme be designed for a range of operating conditions and transitions in between?
- How fast can plant be moved from operating point A to B?
- 2 reactors, 3 distillation columns
- rigorous model including base layer control system: 14.000 DAEs
- 4 controls & 6 path constraints for transition, long time horizon >> 24 hrs



Optimal transition control:

- complexity estimate (single shooting): NLP with 100 Mio embedded DAEs

Industrial Case Study (2)

Computational results: adaptive discretization and parallelization

Discretization of control 3

- Initial guess: 25 parameters
- Adaptive parameterization at final solution: 129 parameters
- Equivalent non-adaptive parameterization: 3072 parameters

→ 95% (or 41 million) equations eliminated by adaptive refinement!

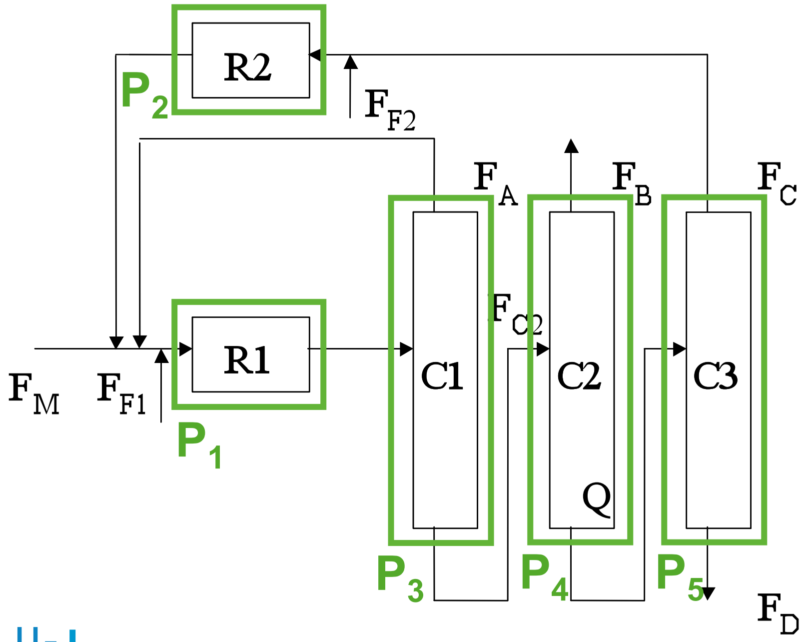
- Calculation time per sensitivity integration: ~ 7500 sec
- Total computation times (adaptive, serial): > 1 month
- Total computation times (adaptive, parallel, 8 CPUs): ~ 1 week

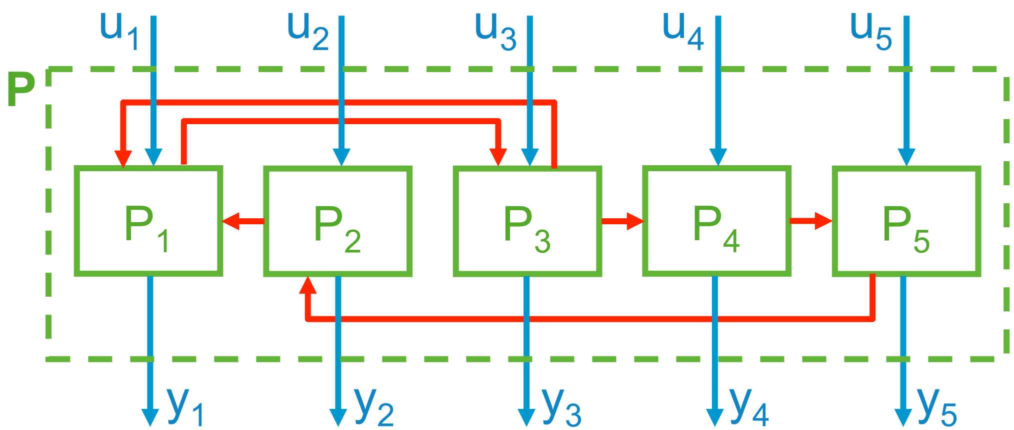
Optimal solution (offline) successful!
Savings of 50 k€ per transition!
Is dynamic real-time optimization feasible?

(Hartwich, Marquardt, 2010)

Control of Process Plants (1)

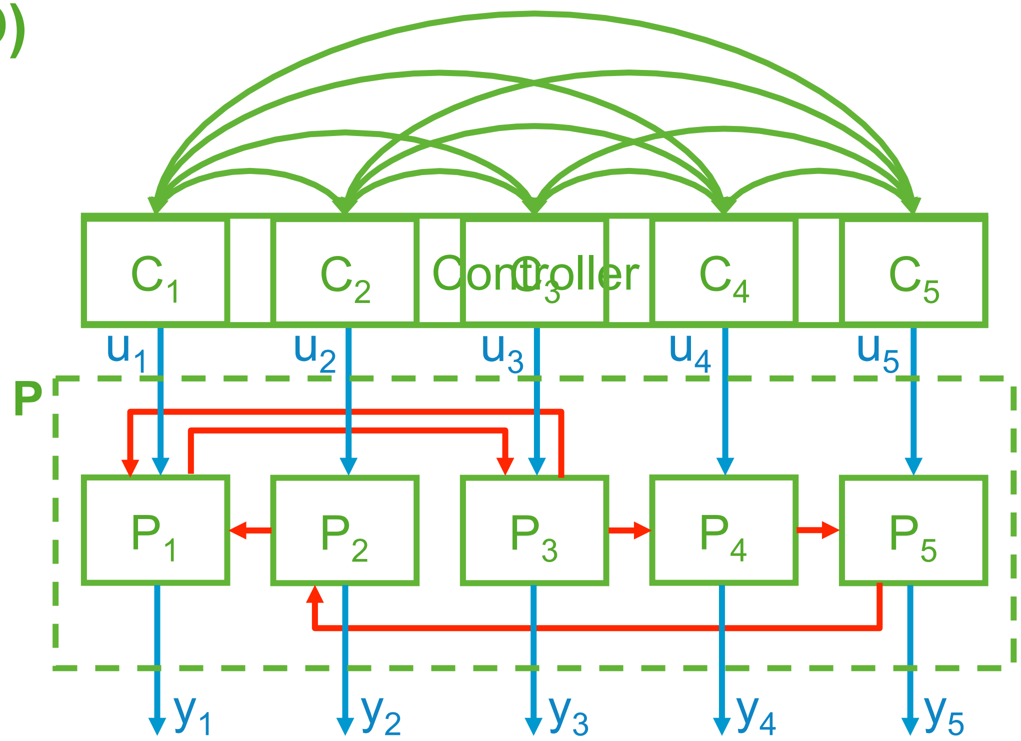
- Process plants can naturally be decomposed into subsystems P_i
  - interconnecting variables: flows, i.e. rate, conc., temp., etc.
  - local inputs: flow rates, etc.
  - local outputs: measurements and interconnecting flows





Control of Process Plants (2)

- **Centralized MPC (or DRTO)**
  - optimal and stable
  - large-scale problem
- **Decentralized MPC (or DRTO)**
  - small-scale problems
  - optimality and stability not guaranteed
- **Distributed MPC (or DRTO)**
  - small-scale problems
  - optimality and stability can be guaranteed (if properly set-up)
  - communication required



(Scattolini, 2009)

Classic Approach – Dual Decomposition (1)

- Consider the convex NLP

$$\min_{p} \sum_{i=1}^{N} \Phi_i(p_i), \qquad p' = [p'_1, ..., p'_N],$$

$$\text{s.t.} \sum_{i=1}^{N} c_i(p_1) \geq 0.$$

- decomposed into subproblems, with primal problems

$$\min_{p_i} \underbrace{\Phi_i(p_i) - \lambda\, c_1(p_i)}_{=L_i(p_i,\lambda)}, \quad \forall i \in 1, ..., N$$

- and dual problem

$$\max_{\lambda} h(\lambda) \overset{def}{=} \min_{p} \sum_{i=1}^{N} L_i(p_i, \lambda)$$

→ **iterate to convergence**

(Lasdon, 1970)

Classic Approach – Dual Decomposition (2)

- Primal problems

$$\min_{p_i} \underbrace{\Phi_i(p_i) - \lambda\, c_1(p_i)}_{=L_i(p_i,\lambda)}, \;\; \forall i \in 1, \dots, N$$

  - cost functions and constraint functions are additive
  - straight forward implementation

- Dual problem

$$\max_{\lambda}\; h(\lambda) \stackrel{def}{=} \min_{p} \sum_{i=1}^{N} L_i(p_i, \lambda)$$

  - **main challenge** for the solution in dual decomposition
  - **normally requires many iterations**

  - convergence can be proven under convexity assumptions

(Lasdon, 1970)

Sensitivity-Driven Decomposition (1)

- Consider a more general NLP:

$$\min_{p} \sum_{i=1}^{N} \Phi_i(p),$$
$$\text{s.t.} \quad c_i(p) \geq 0, \qquad \forall i$$

neither constraints nor objective functions
of subsystems
are additive!

(Scheu and Marquardt 2011a)

Sensitivity-Driven Decomposition (2)

- Consider a more general NLP:

$$\min_p \sum_{i=1}^{N} \Phi_i(p),$$
$$\text{s.t. } c_i(p) \geq 0, \quad \forall i$$

- Parallel iterative solution using decomposed subproblems

$$\min_{p_i} \Phi_i^*(p)$$
$$\text{s.t. } c_i(p) \geq 0,$$

with the strictly convex objective functions

$$\Phi_i^* = \Phi_i(p) + \left[\sum_{\substack{j=1 \\ j \neq i}}^{N} \left. \frac{d\Phi_j}{dp_i} \right|_{p^{[k]}}^T - \lambda_j^{[k]\,T} \left. \frac{dc_j}{dp_i} \right|_{p^{[k]}} \right] (p_i - p_i^{[k]})$$

iterations

(Scheu and Marquardt 2011a)

Why Might this Decomposition Work?

- Let us look at the NCO for the (centralized) NLP

$$\frac{\partial L}{\partial p} = \sum_{i=1}^{N} \left(\frac{\partial \Phi_i}{\partial p} - \lambda_i \frac{\partial c_i}{\partial p} \right) = 0,$$

Condition depends only on first order sensitivities

$$\begin{aligned} c_i(p) &\geq 0, \ \forall i, \\ \lambda_i &\geq 0, \ \forall i, \\ \lambda_i c_i(p) &= 0, \ \forall i. \end{aligned}$$

Directly guaranteed by the subproblems

- Proof of optimality requires comparison of the NCO for the centralized problem and the decomposed problem.

Theorem on Optimality

- Assumptions on centralized NLP:
  - cost functions Φ_i are strictly convex
  - constraint functions c_i are concave

$$\min_p \sum_{i=1}^{N} \Phi_i(p), \quad \text{s.t.} \quad c_i(p) \geq 0, \quad \forall i$$

- Further assumptions
  - p^* solves the centralized NLP and satisfies LICQ
  - distributed algorithm converges and its minimizer satisfies the LICQ

- Then, the minimizer $p^{[k]},\ k \to \infty$, of the distributed problem and the minimizer p^* of the centralized problem are the same, i.e. $\lim_{k\to\infty} p^{[k]} = p^*$.

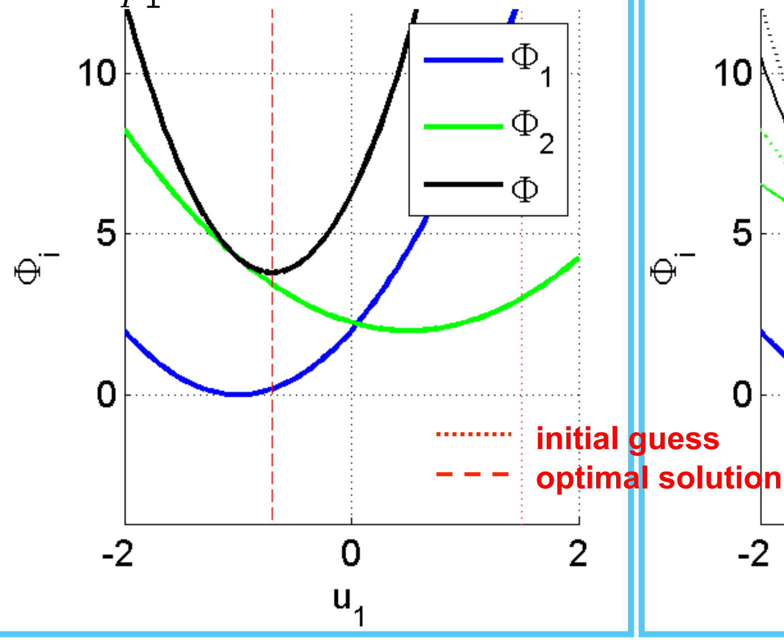
(Scheu and Marquardt 2011a)

Graphical Interpretation

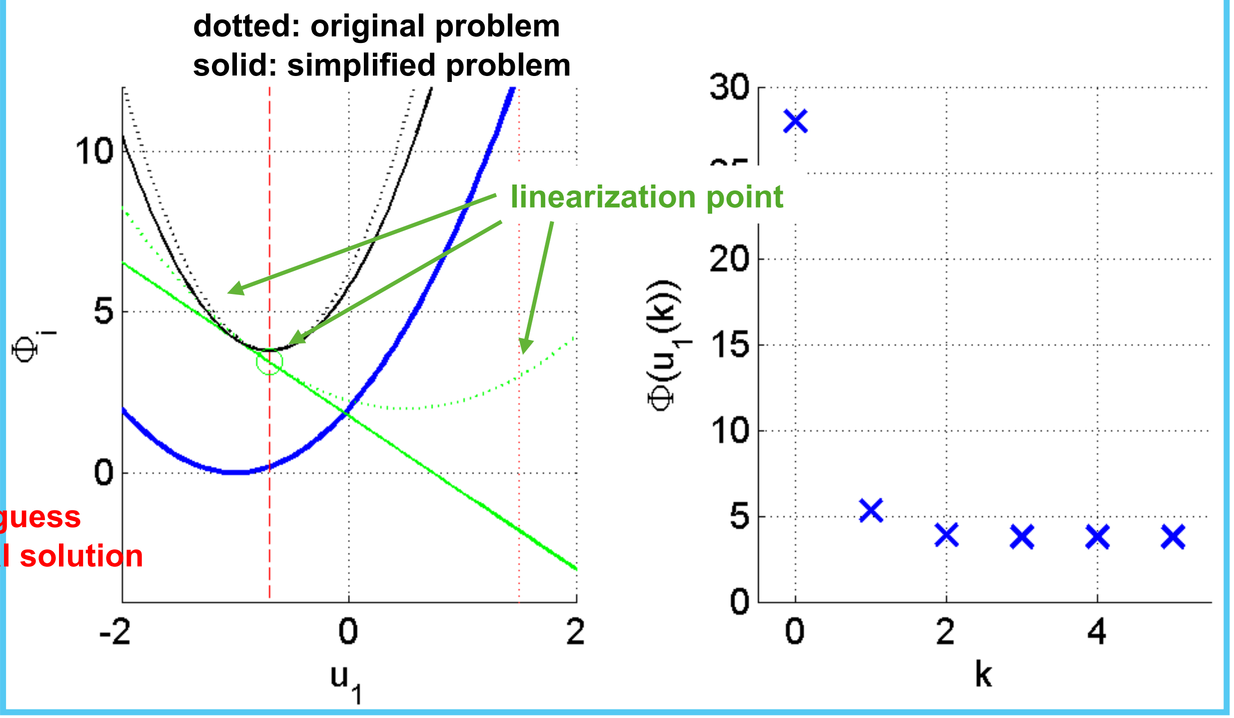
$$\Phi_i^* = \Phi_i + \left[\sum_{\substack{j=1 \\ j \neq i}}^{N} \left. \frac{\mathrm{d}\Phi_j}{\mathrm{d}p_i} \right|_{p^{[k]}} \right] (p_i - p_i^{[k]})$$

overall problem:

$$\min_{p_1} \Phi = 2\Phi_1 + \Phi_2$$



iterative approach



Linear Continuous-Time Systems (1)

- Finite-horizon linear continuous-time optimal control problem:

$$\begin{aligned}\min_{x,u}\ & \frac{1}{2}\int_{t_0}^{t_f}\left(\|x(t)\|_Q^2+\|u(t)\|_R^2\right)dt,\\ \text{s.t. }\ & \dot{x}(t)=Ax(t)+Bu(t), \quad t\in(t_0,t_f],\\ & x(t_0)=x_0,\\ & x\in X, \quad u\in U\end{aligned}$$

- Transcribe into QP

$$\begin{aligned}\min_{p}\ & \sum_{i=1}^{N}\frac{1}{2}p'H_ip+f_i'p\\ \text{s.t. }\ & 0\leq A_ip+b_i, \quad \forall i\end{aligned}$$

(Scheu and Marquardt 2011a)

Sketch of Transcription

1. Discretize the input variables

$$u_{i,j}(t) = \sum_{l} p_{i,j,l} \ \phi_l(t)$$

2. Solve the state variables $x(k)$ for the input parameters p and the initial condition x_0 in discrete time, i.e.

$$x(k) = \mathbf{T}\, p + \mathbf{S}\, x_0$$

3. Transform continuous-time cost function into discrete cost function (Pannocchia et al. 2010)

$$\int_{t_0}^{t_f} x^T \mathcal{Q}_i x + u^T \mathcal{R}_i u d\tau = \sum_{\eta=0}^{\gamma-1} x(\eta)^T \mathcal{Q}^0(\eta) x_(\eta) + p(\eta)^T \mathcal{R}^0(\eta) p(\eta) + 2x(\eta)^T \mathcal{S}^0(\eta) p(\eta)$$

4. Substitute $x(k)$ in the discrete cost function

Linear Continuous-Time Systems (2)

- Transcribe into QP

$$\min_{p} \sum_{i=1}^{N} \frac{1}{2} p' H_i p + f_i' p$$
$$\text{s.t. } 0 \le A_i p + b_i, \quad \forall i$$

$$x(t_0) = x_0,$$
$$x \in X, \quad u \in U$$

- Apply sensitivity-driven decomposition and coordination:

$$\min_{p_i} \Phi_i^* \stackrel{def}{=} \frac{1}{2} \tilde{p}_i^{[k]\ T} \boldsymbol{H}^i \tilde{p}_i^{[k]} + \tilde{p}_i^{[k]\ T} \boldsymbol{f}^i$$
$$+ \left[\sum_{\substack{j=1 \\ j \neq i}}^{N} \left(\left[\boldsymbol{H}_{i1}^j \ \ldots \ \boldsymbol{H}_{iN}^j \right] p^{[k]} + \boldsymbol{f}_i^j - \boldsymbol{A}_i^j \lambda_j^{[k]} \right) \right]^T (p_i - p_i^{[k]}),$$
$$\text{s.t. } c_i(\tilde{p}_i^{[k]}) = \boldsymbol{A}^{i\ T} \tilde{p}_i^{[k]} + \boldsymbol{b}^i \ge 0$$

(Scheu and Marquardt 2011a)

Convergence Analysis

- Algorithm defines a fixed point iteration method, analysis based on the KKT NCO

$$\begin{bmatrix} p^{[k+1]} \\ \lambda^{[k+1]} \end{bmatrix} = - \begin{bmatrix} \boldsymbol{H}_{\text{diag}} & -\boldsymbol{A}_{\text{diag}} \\ -\boldsymbol{A}_{\text{diag}}^T & 0 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{j=1}^{N} \boldsymbol{H}^j & -\boldsymbol{A} \\ -\boldsymbol{A}^T & 0 \end{bmatrix} \begin{bmatrix} p^{[k]} \\ \lambda^{[k]} \end{bmatrix} + \begin{bmatrix} p^{[k]} \\ \lambda^{[k]} \end{bmatrix}$$
$$- \begin{bmatrix} \boldsymbol{H}_{\text{diag}} & -\boldsymbol{A}_{\text{diag}} \\ -\boldsymbol{A}_{\text{diag}}^T & 0 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{j=1}^{N} \boldsymbol{f}^j \\ -\boldsymbol{b} \end{bmatrix}$$

- Small-gain theorem can be applied, convergence for

$$L = \left\| \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} - \begin{bmatrix} \boldsymbol{H}_{\text{diag}} & -\boldsymbol{A}_{\text{diag}} \\ -\boldsymbol{A}_{\text{diag}}^T & 0 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{j=1}^{N} \boldsymbol{H}^j & -\boldsymbol{A} \\ -\boldsymbol{A}^T & 0 \end{bmatrix} \Big|_{\mathcal{A}} \right\| < 1$$

(Scheu and Marquardt 2011a)

Enforce Convergence

- Further modification of the cost function

$$\Phi_i^{+} = \Phi_i^* + \frac{1}{2}(p_i - p_i^{[k]})'\Omega_i(p_i - p_i^{[k]})$$

- → constant L does also depend on $\mathbf{\Omega_i}$:

$$L = \left\| \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} - \begin{bmatrix} \boldsymbol{H}^{\Omega}_{\text{diag}} & -\boldsymbol{A}_{\text{diag}} \\ -\boldsymbol{A}^T_{\text{diag}} & 0 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{j=1}^{N} \boldsymbol{H}^j & -\boldsymbol{A} \\ -\boldsymbol{A}^T & 0 \end{bmatrix} \Big|_{\mathcal{A}} \right\| < 1$$

- gradient-free optimization (Wegstein, 1958; Westerberg et al., 1979)
- generalization of proximal minimization algorithm (Rockafellar 1976; Censor 1992)

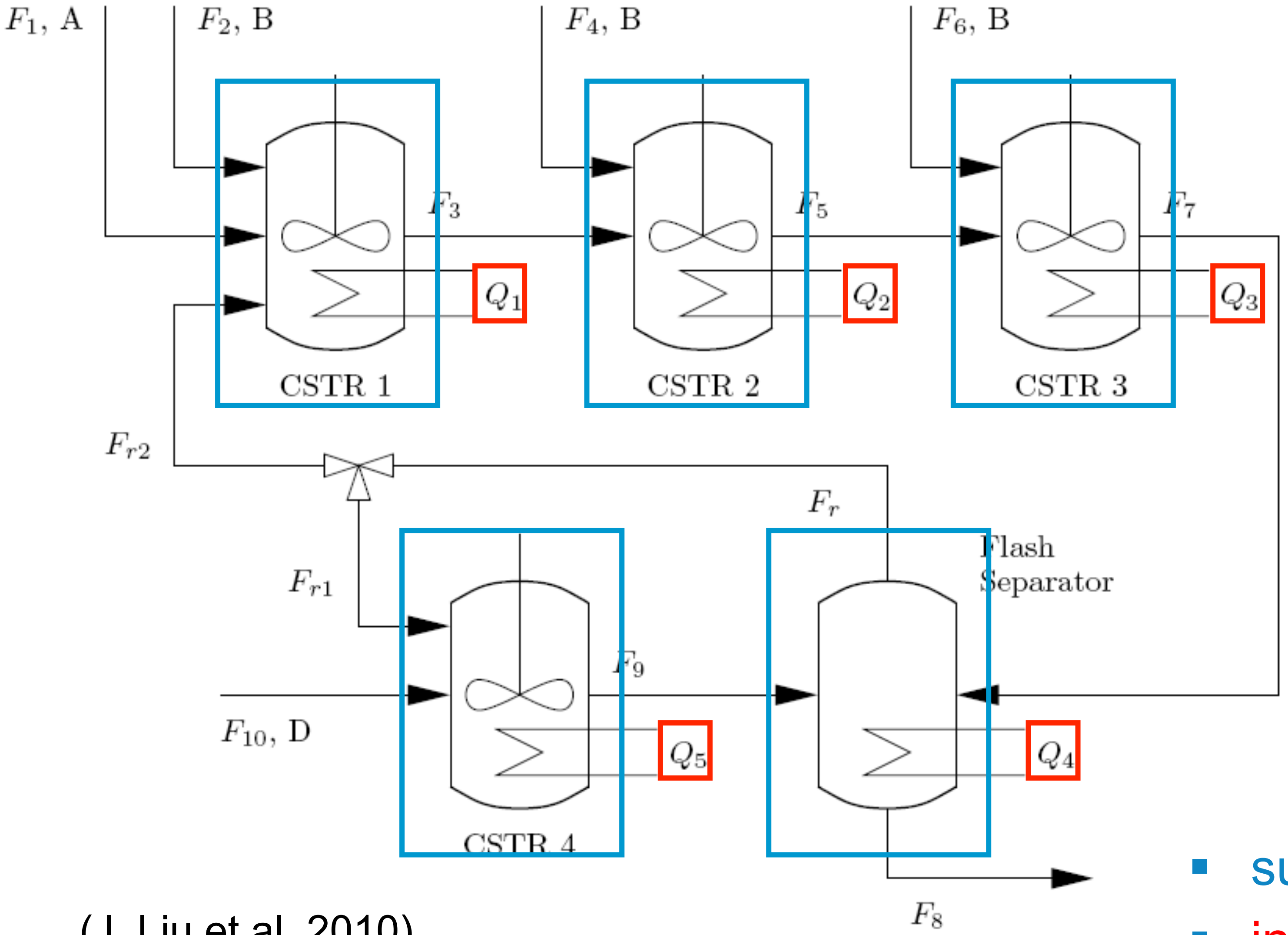
Sensitivity-Driven Distributed MPC (S-DMPC)

In closed loop, do on each horizon:

1. Measure or estimate the current system state.
2. Transcribe the optimal control problem into QP.
3. Select
   - initial parameters $p^{[0]}(h)$ and
   - initial Lagrange multipliers $\lambda^{[0]}(h)$.
   - Warm start based on preceding horizon.
4. Apply the distributed QP algorithm described before.
5. Apply the calculated optimal control inputs $u_{i,j}(t) = \sum_l p_{i,j,l} \; \phi_l(t)$ to the plant.

coooperative, iterative, optimal on convergence, neighbor-to-neighbor communcation

Illustrative Example – Alkylation of Benzene



(J. Liu et al. 2010)

- subsystems
- inputs

Sketch of Mathematical Model

For each subsystem:

- Mass balances for each species and energy balance

$$\begin{bmatrix} \frac{\mathrm{d}\, c_{Ai}}{\mathrm{d}t} \\ \frac{\mathrm{d}\, c_{Bi}}{\mathrm{d}t} \\ \frac{\mathrm{d}\, c_{Ci}}{\mathrm{d}t} \\ \frac{\mathrm{d}\, c_{Di}}{\mathrm{d}t} \\ \frac{\mathrm{d}\, T_i}{\mathrm{d}t} \end{bmatrix} = f_i(\ldots)$$

"Medium-scale" DAE system:
- 25 differential equations
- ~100 algebraic equations

For stirred tank reactors:

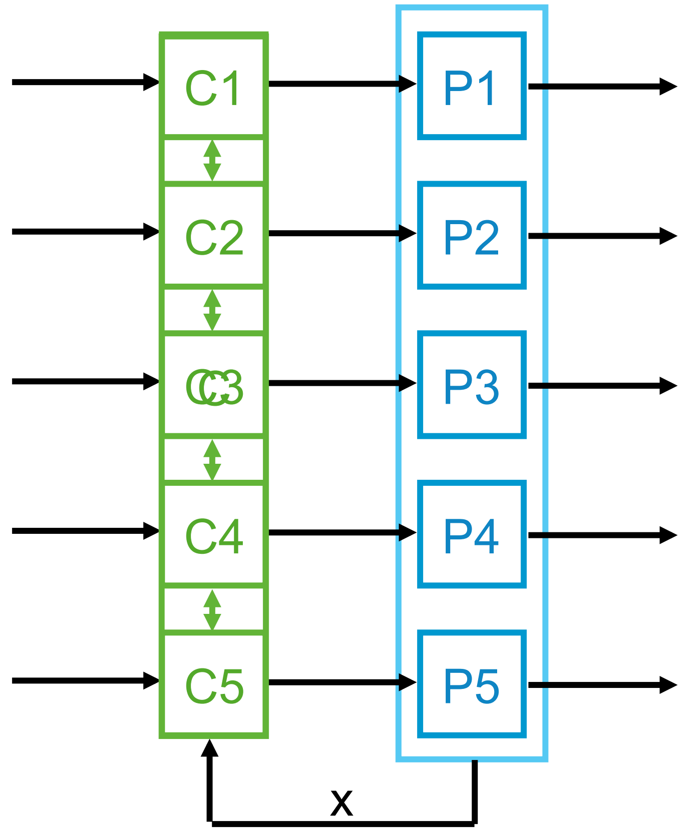
- nonlinear reaction kinetics

For flash separator:

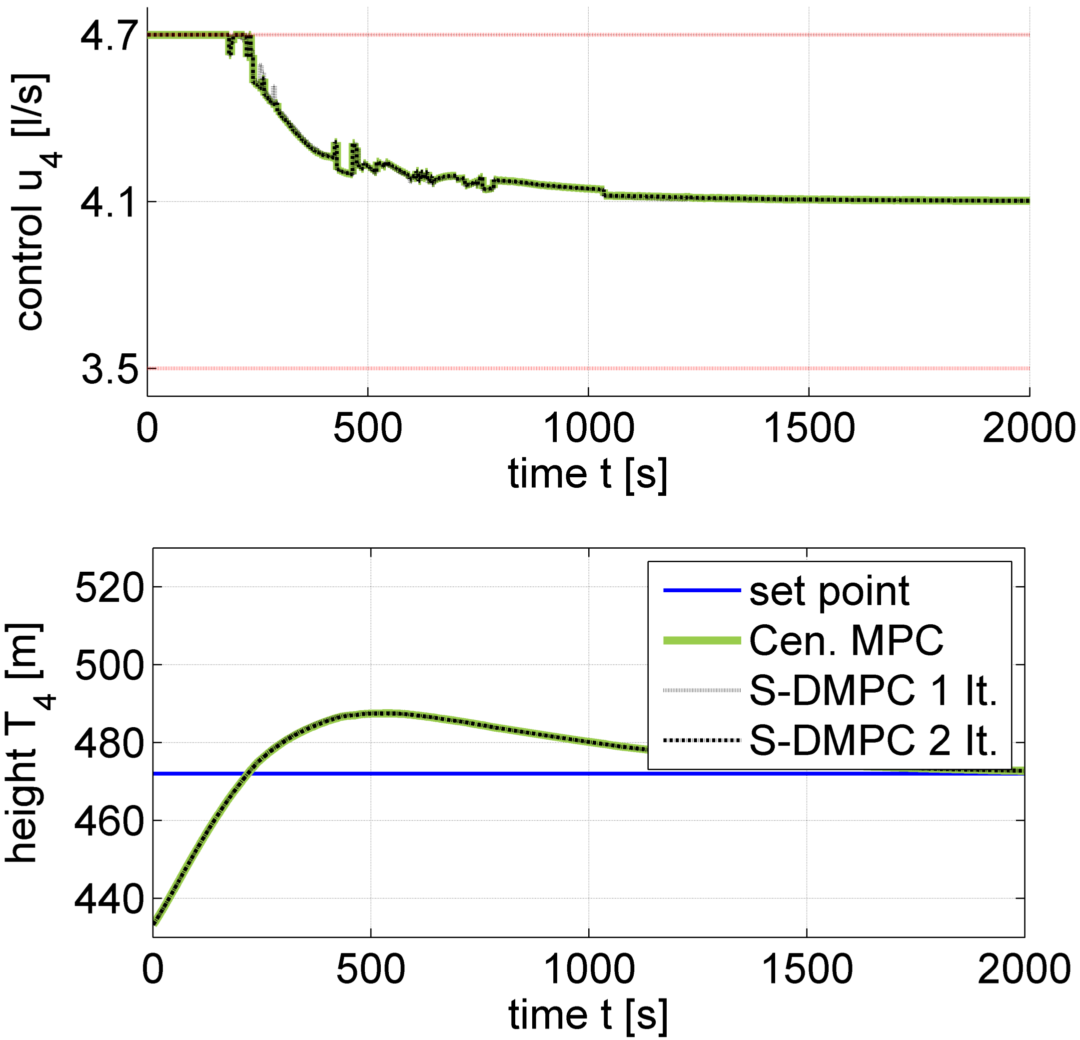
- nonlinear phase equilibrium and physical property models

Sketch of Controller Design

- Nonlinear process model
- Full state feedback
- Linear controller, based on linearization of nonlinear model
  - centralized
  - distributed
- no further disturbances, but plant-model mismatch
- set-point tracking



Results



S-DMPC provides the same controller performance as a centralized MPC

Solve 5 small QP in parallel instead of 1 large QP

→ faster computation possible

(Scheu and Marquardt, 2011a)

Linear Discrete-Time Systems

- Finite horizon discrete-time linear optimal control problem:

$$\min_{x,u} \frac{1}{2} \sum_{k=k'}^{k'+K-1} \left(\|x(k)\|_Q^2 + \|u(k)\|_R^2 \right) + \|x(h)\|_P^2,$$

$$\text{s.t.} \quad x(k+1) = Ax(k) + Bu(k), \qquad k = k', \ldots, k'+K-1,$$

$$x(k') = x_{k'},$$

$$x(k) \in X, u(k) \in U$$

- Write as QP

$$\min_{p} \sum_{i=1}^{N} \frac{1}{2} p' H_i p + f_i' p$$

$$\text{s.t.} \quad 0 \leq A_i p + b_i, \qquad \forall i,$$

$$0 = A_i^{eq} p + b_i^{eq}, \qquad \forall i$$

- Apply sensitivity-driven coordination

(Scheu & Marquardt 2011b)

Continuous-time vs. discrete-time

Continuous-time

- also possible for higher order input representations
- non-uniform control-grid possible
- system couplings are solved during transcription
- couplings could also be included in finite number of equality-constraints
- most natural for nonlinear case

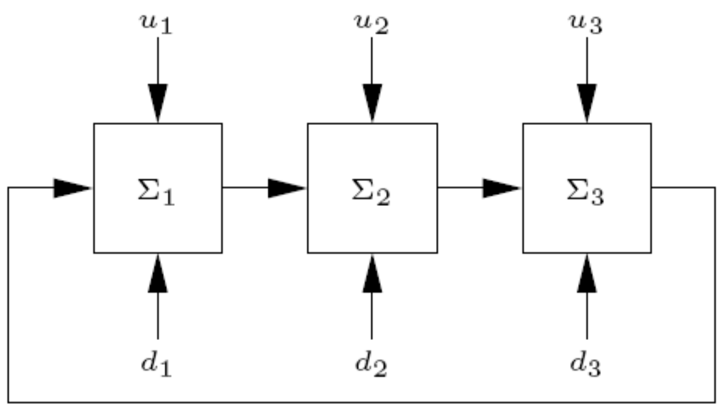
Discrete-time

- only piecewise constant inputs
- uniform control-grid
- system couplings are included in equality-constraints
- couplings could also be solved by transcription
- difficult to extent to nonlinear cases

Case Study

- Discrete-time linear system with unknown disturbances

$$x(k+1) = A\ x(k) + B\ u(k) + D\ d(k)$$

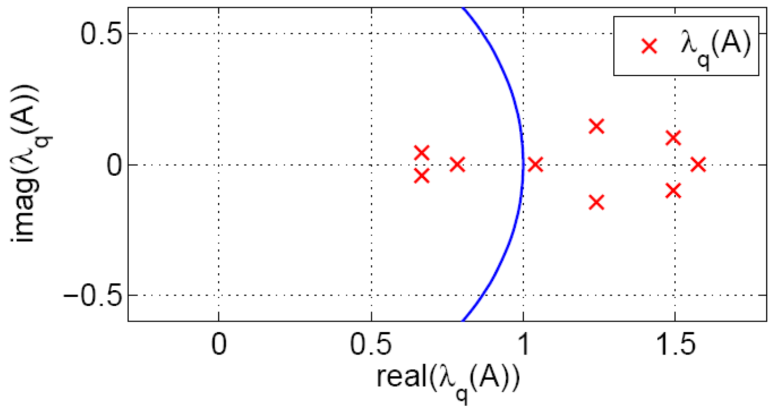


- where

$$A = \left[\begin{array}{c|c|c} A_{11} & 0 & A_0 \\ \hline A_0 & A_{22} & 0 \\ \hline 0 & A_0 & A_{33} \end{array}\right]$$

$$B = D = \left[\begin{array}{c|c|c} B_0 & 0 & 0 \\ \hline 0 & B_0 & 0 \\ \hline 0 & 0 & B_0 \end{array}\right]$$

- 9 differential state variables
- 3 scalar inputs
- 3 scalar disturbances
- unstable system dynamics:



(Scheu and Marquardt 2011b)

MPC Setup

- Centralized MPC – 1 monolithic controller with full system knowledge, **large QP**
- Decentralized MPC – 3 independent controllers, **small QP**
- Dual Decomposition – 3 low layer controller, 1 coordinator, **small QP**
- S-DMPC – 3 cooperative controllers, **small QPs**

- Disturbances

$$d_1(k) = \begin{cases} 0.1, & \text{for } 75 \leq k \leq 150 \\ 0, & \text{else} \end{cases},$$

$$d_2(k) = \begin{cases} 0.1, & \text{for } 225 \leq k \leq 300 \\ 0, & \text{else} \end{cases},$$

$$d_3(k) = \begin{cases} 0.1, & \text{for } 375 \leq k \leq 450 \\ 0, & \text{else} \end{cases}$$

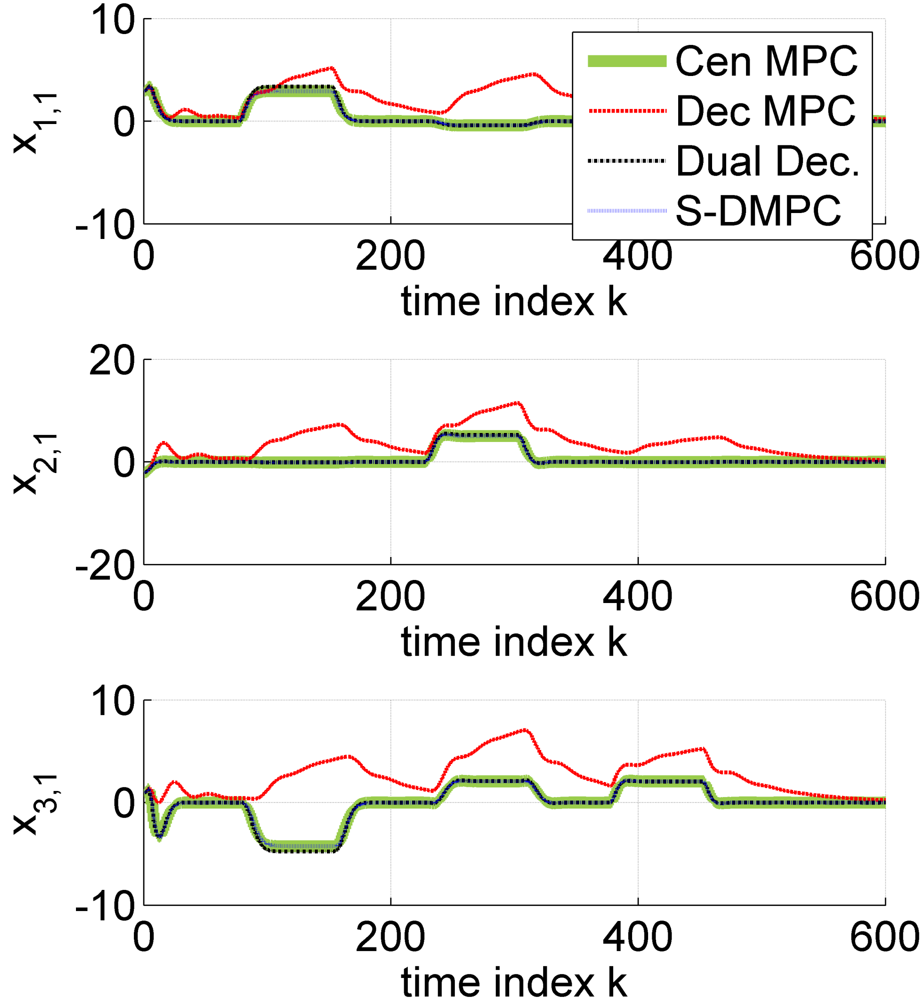
MPC Setup (cont.)

- no terminal cost
- long prediction and control horizon (K = 50)
- solved using Matlab standard QP solver **`quadprog`** with standard settings
- J = 30 iterations required for dual decomposition approach for convergence
- J = 1 and J = 2 iterations for S-DMPC → low communication and computing requirements

Closed-loop Trajectories

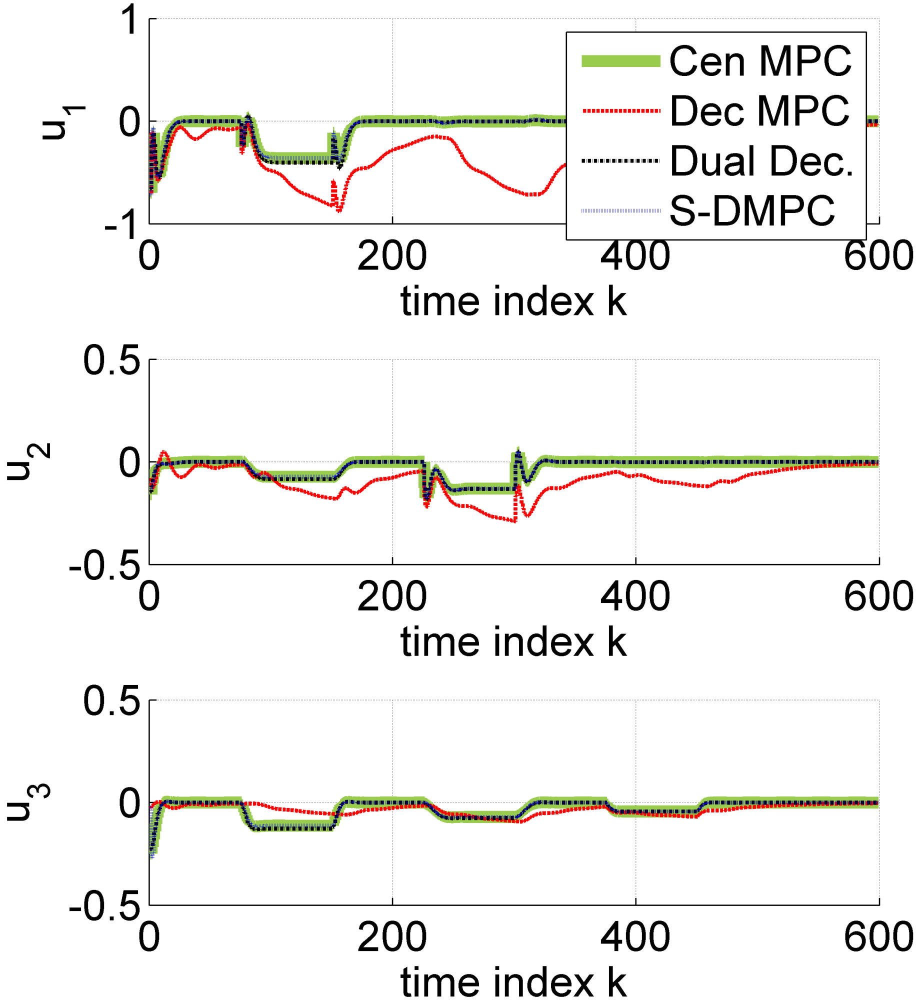
State trajectories:

- Decentralized MPC
  - bad disturbance compensation
  - almost unstable control
- Dual Decomposition
  - achieves good performance
  - requires many iterations (here 30)
- S-DMPC
  - only one iteration
  - almost matches the centralized control



Closed-loop trajectories

Input trajectories



Controller Performance

- Absolute performance (quadratic performance index)

$$\Phi_{\text{abs}} = \sum_{k=0}^{H-1} \left(\|x(k)\|_Q^2 + \|u(k)\|_R^2 \right)$$

- Relative performance (Centralized controller is reference)

$$\Phi_{\text{rel}} = \frac{\Phi_{\text{abs}} - \Phi_{\text{abs,ref}}}{\Phi_{\text{abs,ref}}}$$

- Simulation results

Method	It.	Φ_{abs}	Φ_{rel} [%]
Cen. MPC	–	1.94e4	–
Dec. MPC	–	1.34e5	589
Dual Dec.	30	2.30e4	18.6
S-DMPC	1	1.95e4	0.5
S-DMPC	2	1.94e4	0

Computing Time

- Comparison of average computing time for the methods considered

Method	It.	$\bar{t}$ [s]
Cen. MPC	–	0.112
Dec. MPC	–	3×0.026
Dual Dec.	30	3×0.922
S-DMPC	1	3×0.030
S-DMPC	2	3×0.059

- Computing time can be reduced, in particular with multiple CPU cores

- Dual decomposition is not competitive

Conclusions & Future Work

Conclusions

- S-DMPC: a new method for distributed optimal control
  - inherits properties of centralized optimal control problem
  - S-DMPC provides optimal performance
- S-DMPC enables distributed computing
  - size of QP to be solved reduced
  - computing time can be reduced

Future work

- guaranteed stability (e.g. infinite horizon, terminal constraint, …)
- output feedback
- convergence (adaptation of QP via Wegstein extension)
- nonlinear systems
- Efficient implementation and integration
  into dynamic real-time optimization platform of AVT.PT

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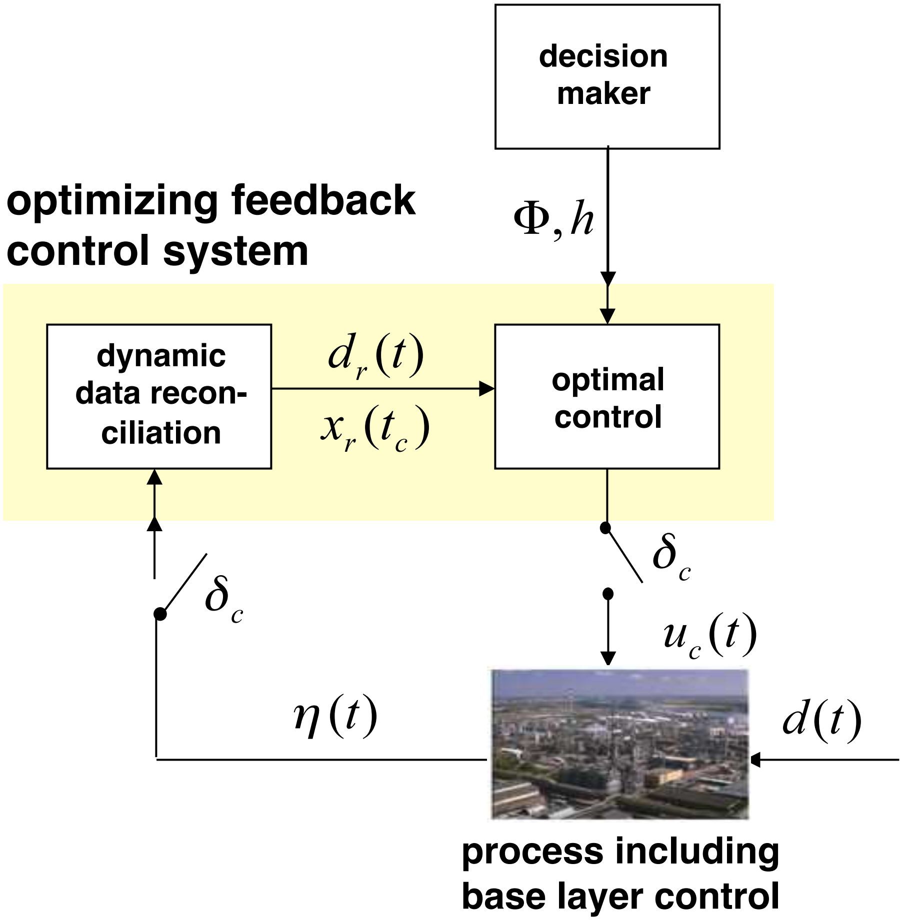
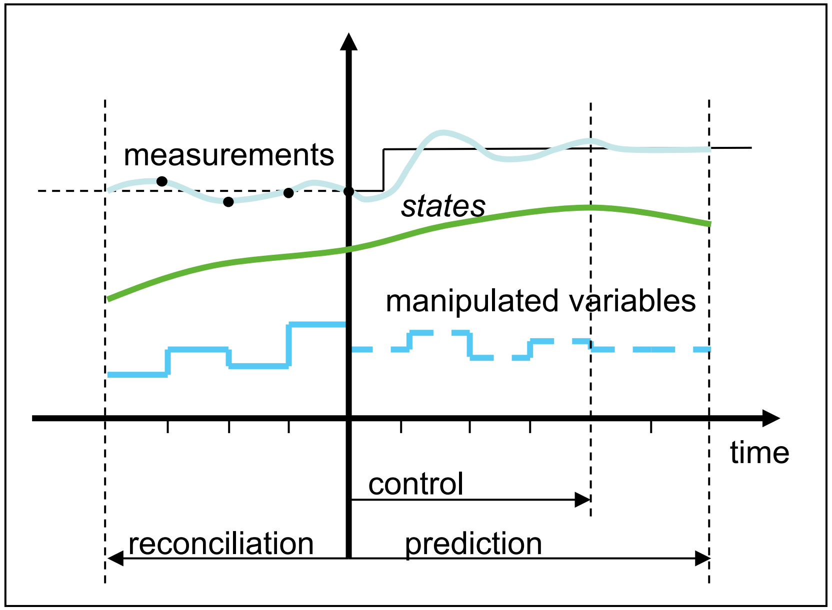
AACHENER VERFAHRENSTECHNIK

Backup

Multi Layer Model-Predictive Control

Dynamic Real-Time Optimization



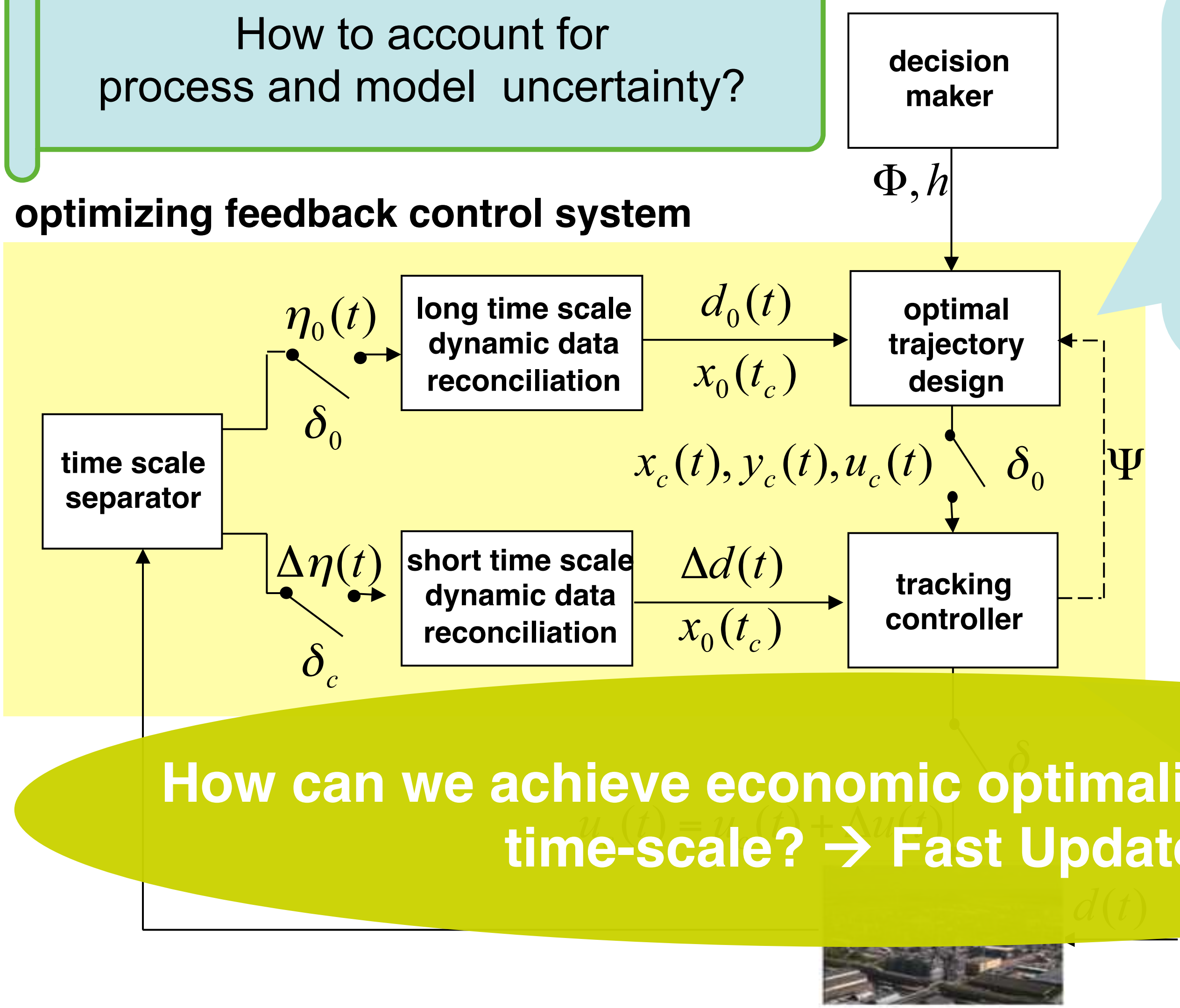


- economical objectives & constraints
- optimal output feedback
- solution of optimization problems at sampling frequency
- computationally demanding, limited by model complexity

Time-Scale Decomposition

How to account for
process and model uncertainty?

optimizing feedback control system



slow time-scale
- changing environment, process variations …
- most economical trajectory satisfying safety or equipment constraints!

fast time-scale
- measurement and process noise …
- trajectory tracking satisfying control

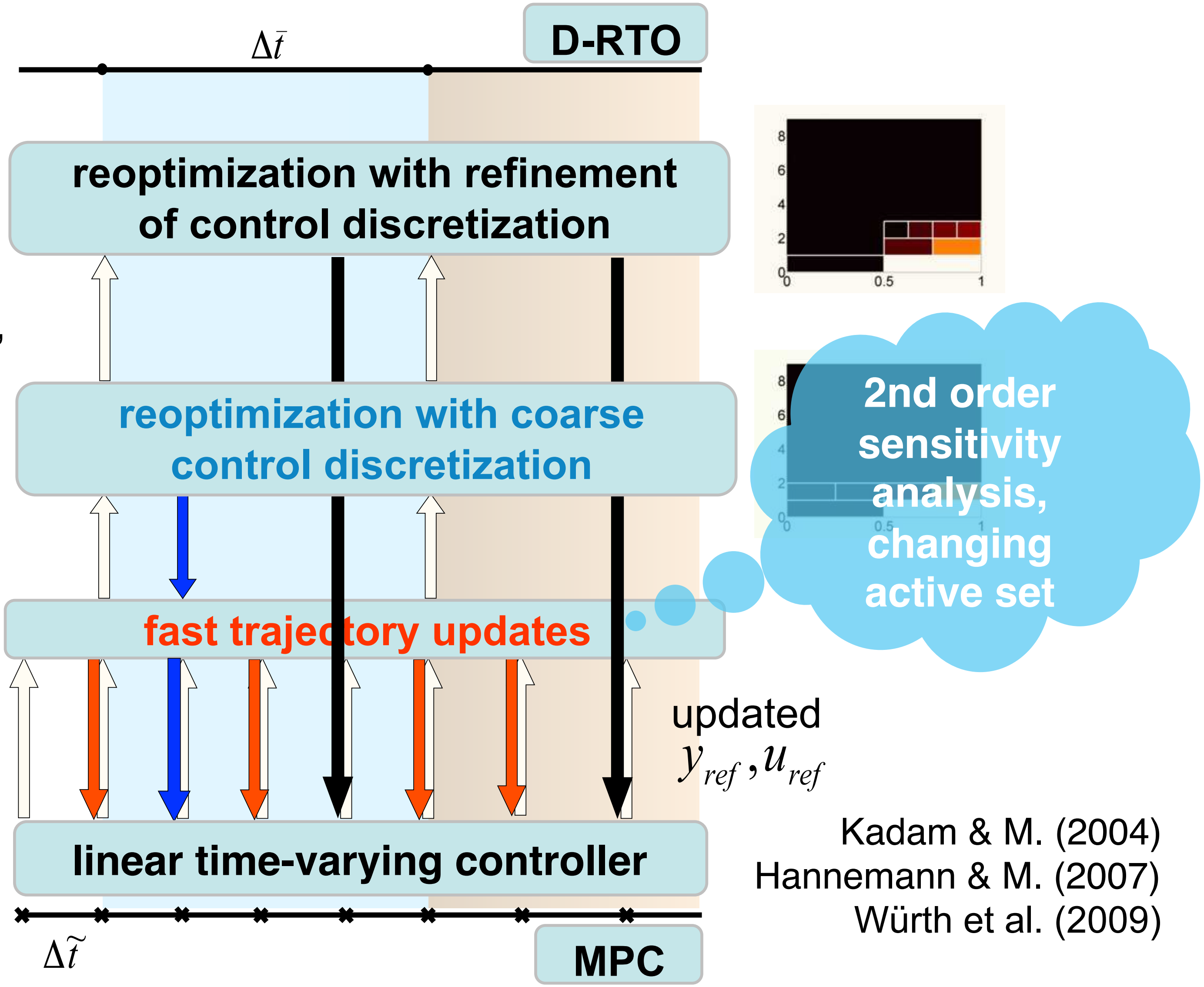
How can we achieve economic optimality on the faster time-scale? → Fast Updates

Integration of Control and Optimization

dynamic real-time optimization (D-RTO), trajectory updates when necessary

neighboring extremal update, when possible

linear time-varying MPC in delta-mode for trajectory tracking,



Kadam & M. (2004)
Hannemann & M. (2007)
Würth et al. (2009)

Fast Neighboring Extremal Updates

(Kadam & Marquardt, 2004;
Würth et al., 2009)

- parameterize uncertainty
- exploit sensitivity information of previously solved optimization problem to generate an approximation of the optimal update

Sensitivity system (Fiacco, 1983), invariant active set

$$\begin{bmatrix} L_{pp}(\cdot) & -g_p^{a,T}(\cdot) \\ g_p^{a,T}(\cdot) & 0 \end{bmatrix} \begin{bmatrix} p_\theta \\ \lambda_\theta \end{bmatrix} = -\begin{bmatrix} L_{p\theta}(\cdot) \\ g_\theta(\cdot) \end{bmatrix}$$

$$\Delta p := p(\theta) - p_0 = p_\theta(\theta_0)\Delta\theta$$
$$\Delta\lambda^a := \lambda^a(\theta) - \lambda_0^a = \lambda_\theta^a(\theta_0)\Delta\theta$$
$$\Delta\lambda^{ina} := \lambda^{ina}(\theta) - \lambda_0^{ina} = 0$$

L: Lagrange function
f: objective function
g: constraints
p: discretized controls
θ*: uncertain param.*

Changing active set (Ganesh & Biegler, 1987)

$$\min_{\Delta z} \; 0.5\Delta p^T L_{pp}^{ref}\Delta p + \Delta\theta^T L_{p\theta}^{ref}\Delta p + f_p^{T,ref}\Delta p$$
$$\text{s.t.} \quad g_p^{ref}\Delta p \geq -g_\theta^{ref}\Delta\theta + g^{ref}$$

- compute first- and second-order derivatives
- solve QP for fast update
- re-iterate if necessary

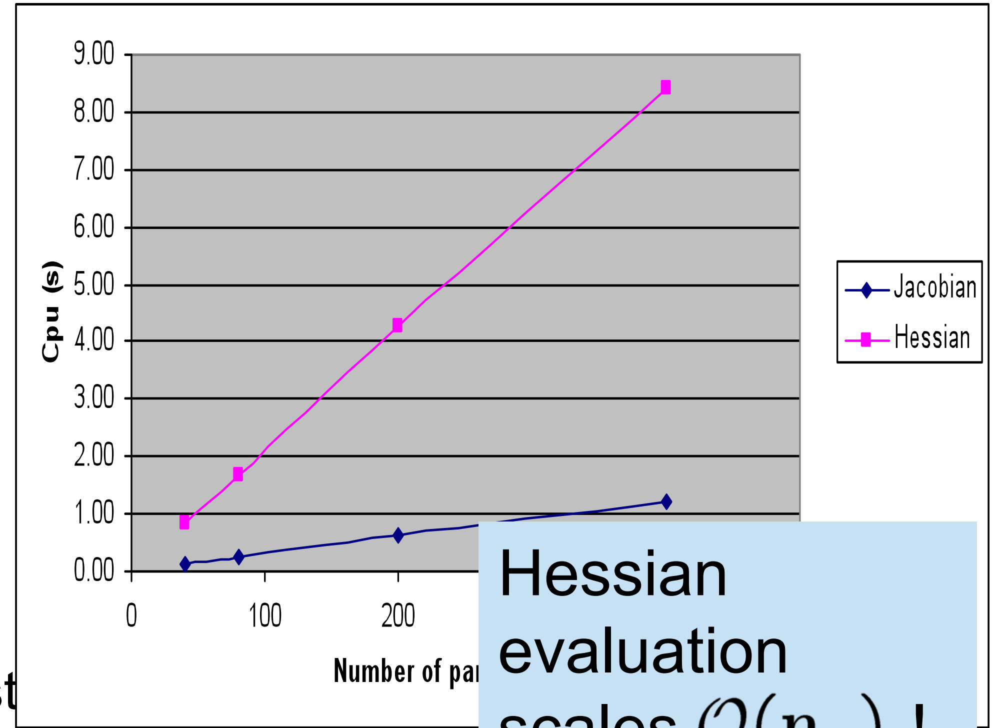
$L_{pp}, L_{p\theta}, f_p, g_p, g_\theta$

Efficient Computation of 2nd order Sensitivities

- finite differences and 2nd order forward sensitivities (Vassiliadis et al., 1999) scale $O(n_p^2)$
- adjoint sensitivity analysis for problems without path constraints (Cao et al., 2003, Özyurt et al., 2005)
- 2nd order adjoint sensitivity analysis for path-constrained problems (Hannemann & M., 2007, 2010) → **NIXE**

➡ Superposition principle for the linear adjoint system: only one 2nd order adjoint syst

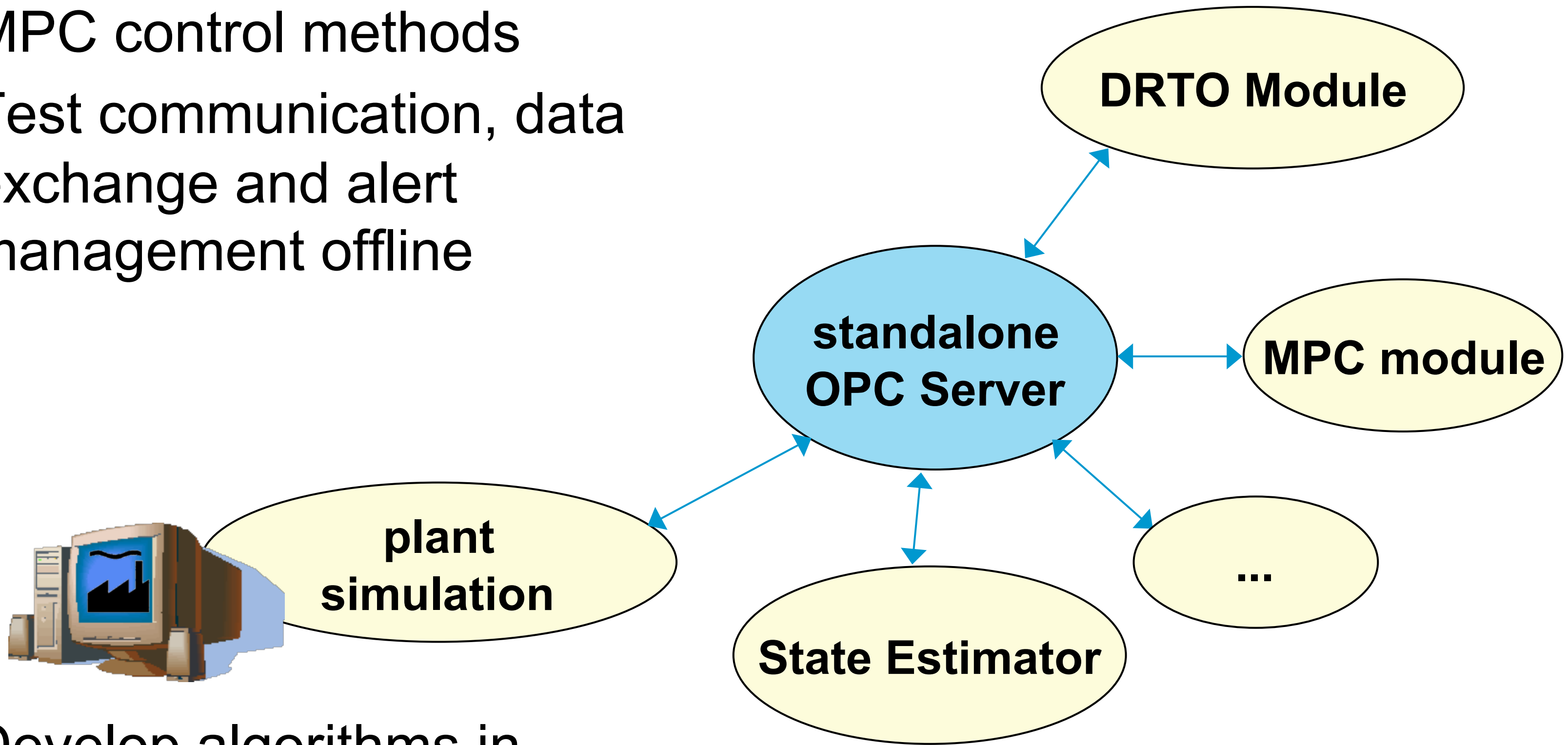
Williams-Otto benchmark problem



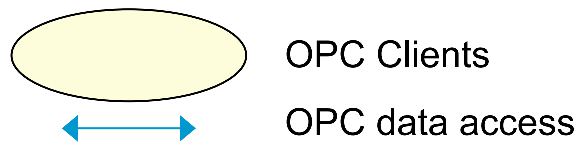
Hessian evaluation scales $\mathcal{O}(n_p)$!

Software Realization – DRTO Toolbox (1)

- Use plant simulator for development of advanced MPC control methods
- Test communication, data exchange and alert management offline

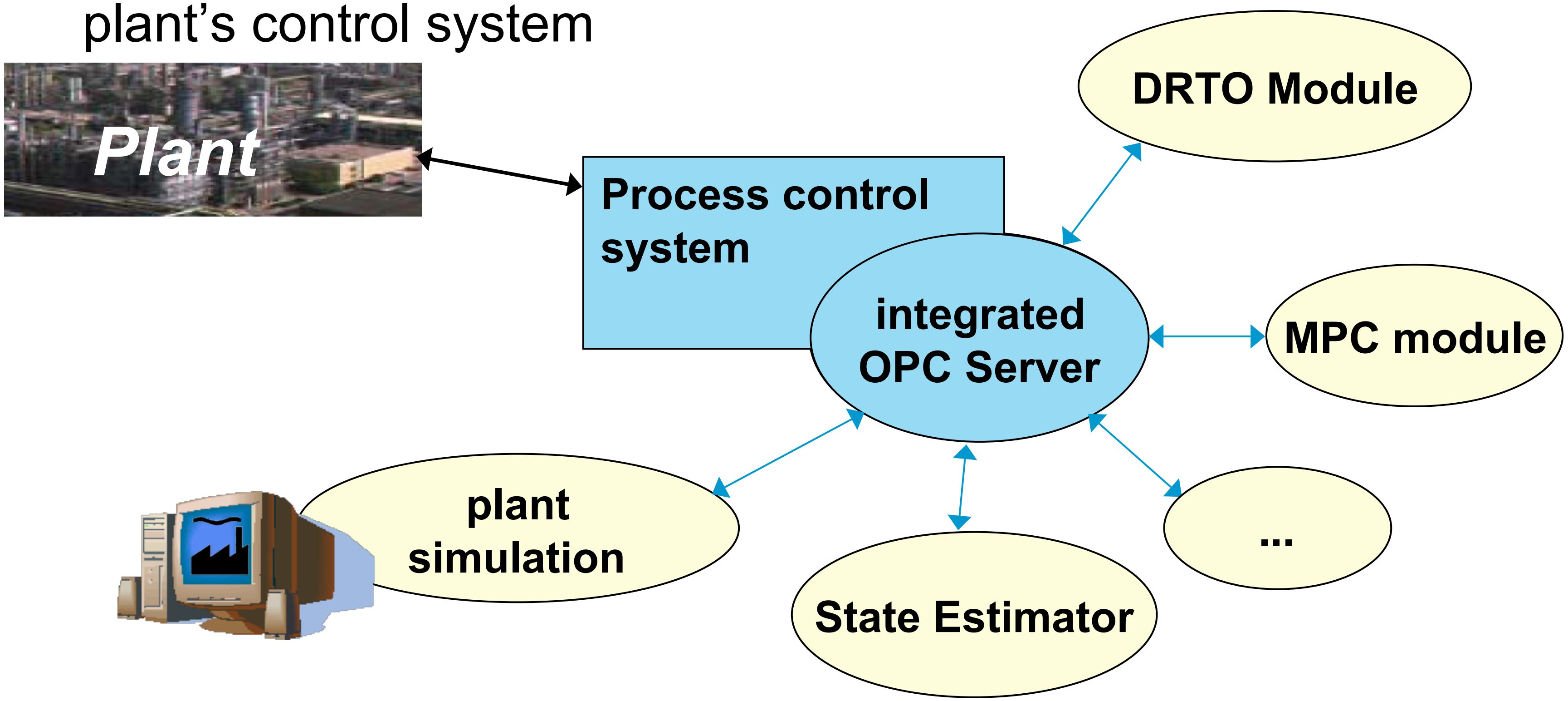


- Develop algorithms in Matlab, gPROMS, C++, ...

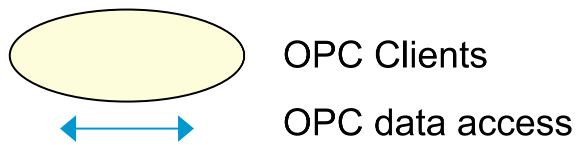


Software Realization – DRTO Toolbox (2)

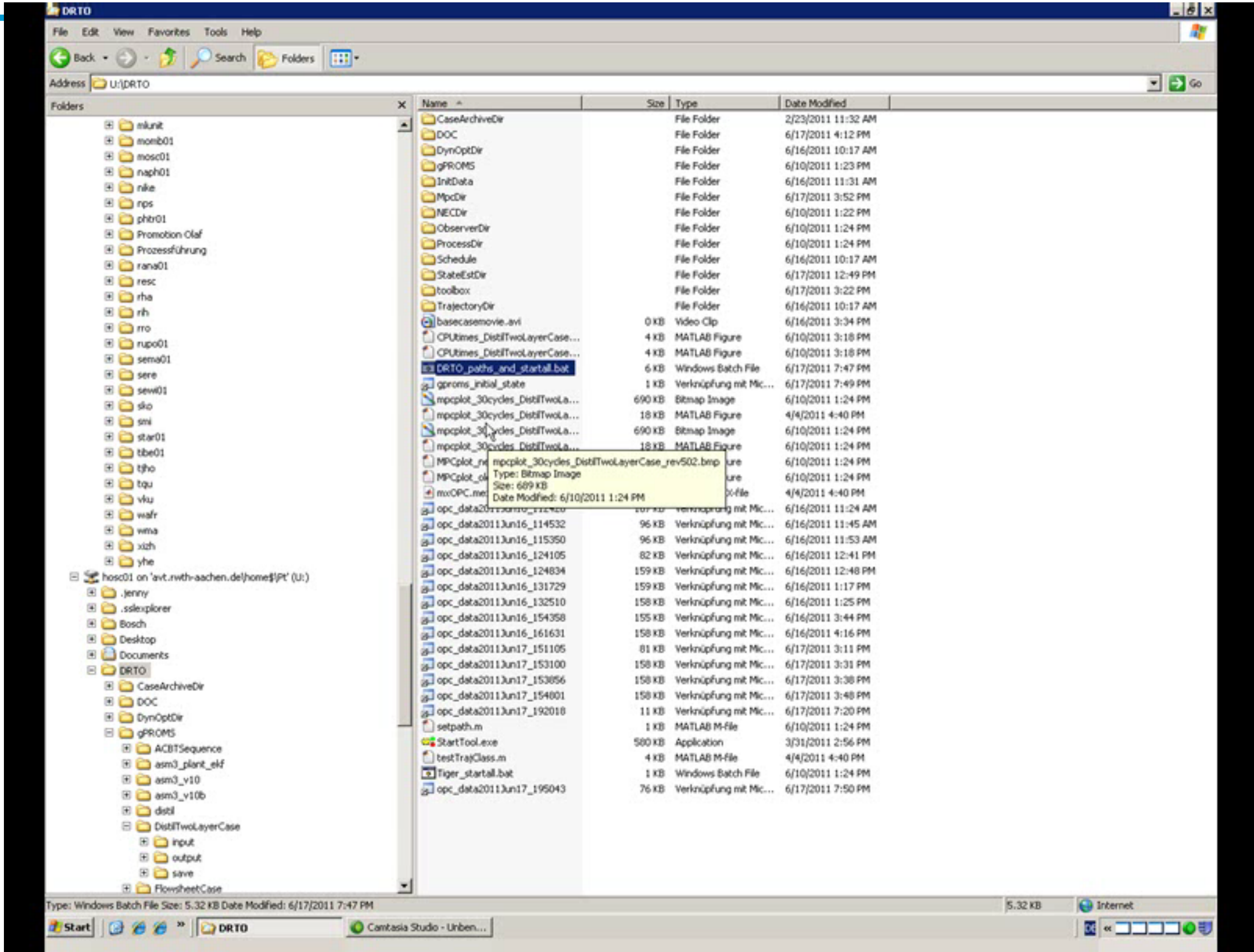
- Connect the control methods to the real control process through the plant's control system



- (Data structure in plant simulation should be the same as in PCS)

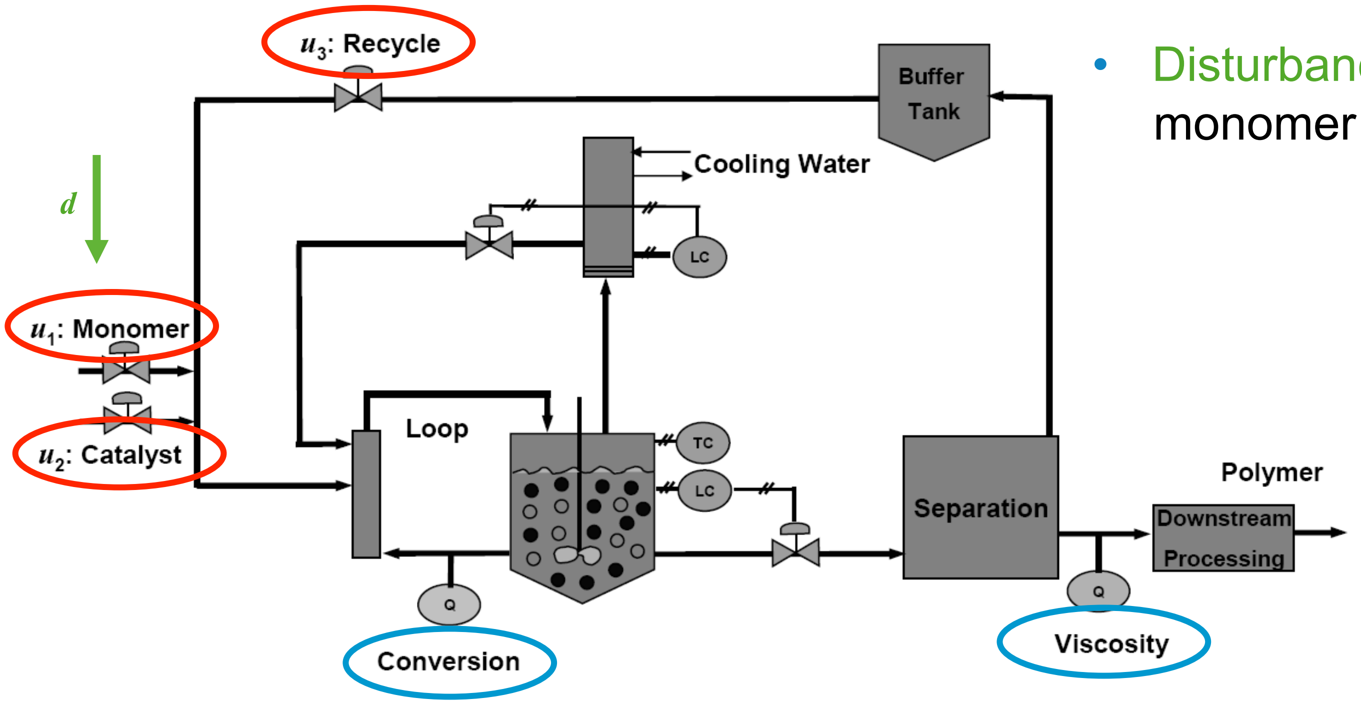


Software Realization – DRTO Toolbox (3)



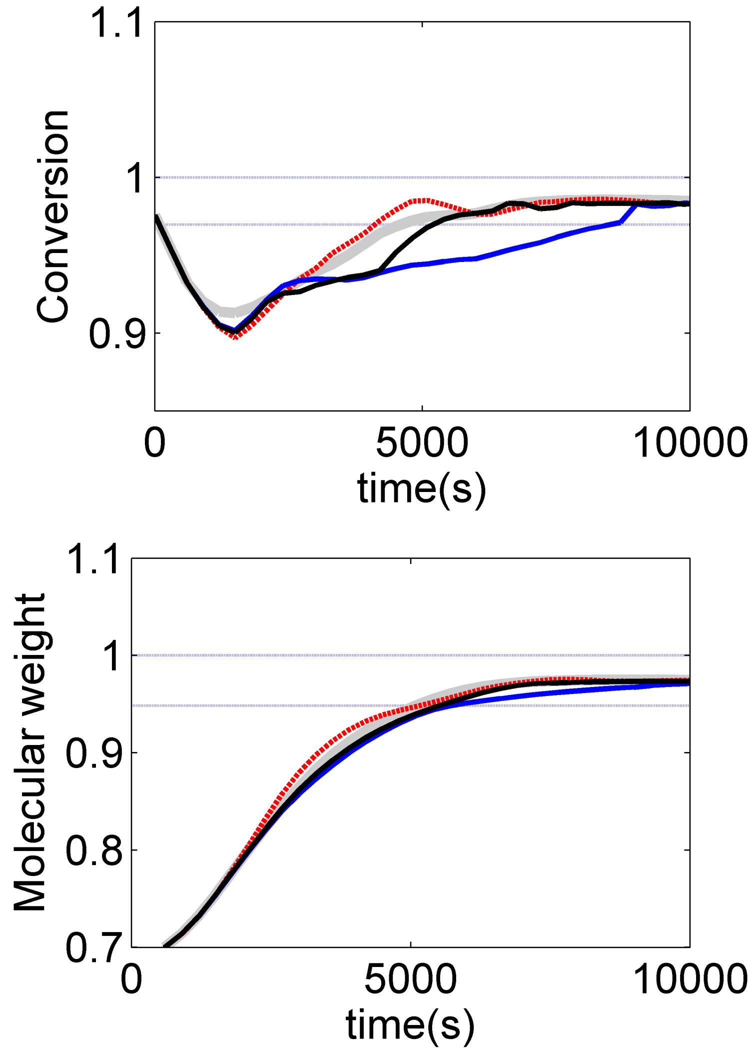
Case Study – Continuous Polymerization Process (1)

- Large-scale industrial process (Bayer AG, Dünnebier et al., 2004)
  - ~ 200 (dynamic) state variables
  - ~ 2000 algebraic variables
  - 3 manipulated variables
  - Task: Set point change from polymer A to B



- Disturbance: Ratio of monomer 1 and monomer 2

Case Study – Continuous Polymerization Process (2)



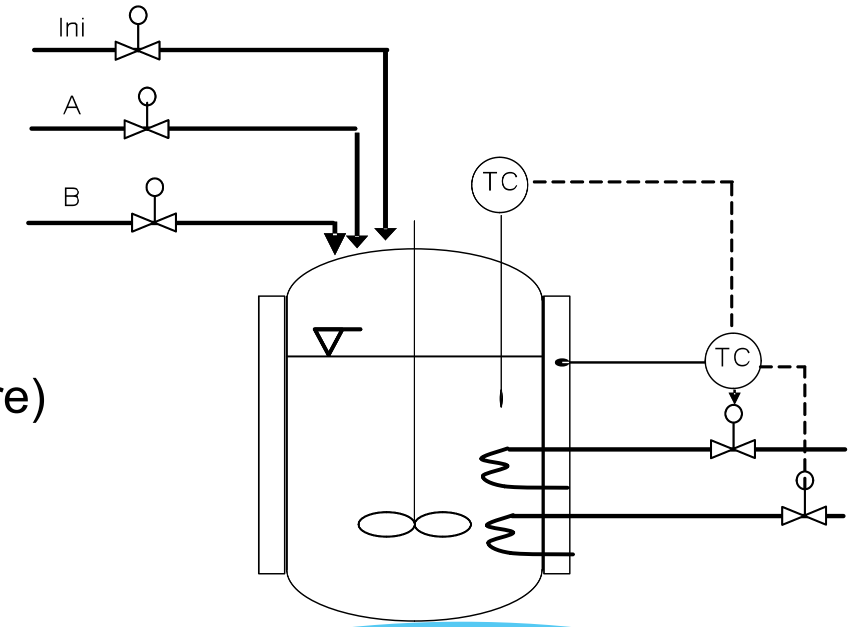
- Reference control strategy
  - Objective value: 0.59
  - Constraint violations: 1.6
- Delayed Single-Layer DRTO
  - Objective value: 1.18
  - Constraint violations: 16.2
- Single Layer: Neigboring Extremal Updates (NEU)
  - Objective value: 0.74
  - Constraint violations: 2.0
- Two-Layer (DRTO and NEU)
  - Objective value: 0.61
  - Constraint violations: 2.1

(Würth et al., 2011)

Experimental Evaluation in an Industrial Setting

- **Semi-batch polymerization (BASF)**
  - co-polymer of styrene & butyl-acrylate
  - solution polymerisation
  - complex reaction kinetics
  - detailed heat transfer model
- **Control objectives**
  - minimization of batch time and simultaneous optimization of profit
  - disturbance rejection (feed pump failure)
  - endpoint polymer quality constraints
- **Dynamic model (from literature)**
  - 250 DAEs, 4 controls, 6 constraints
  - 5 experiments for parameter identification (heat transfer, viscosity, reaction kinetics)

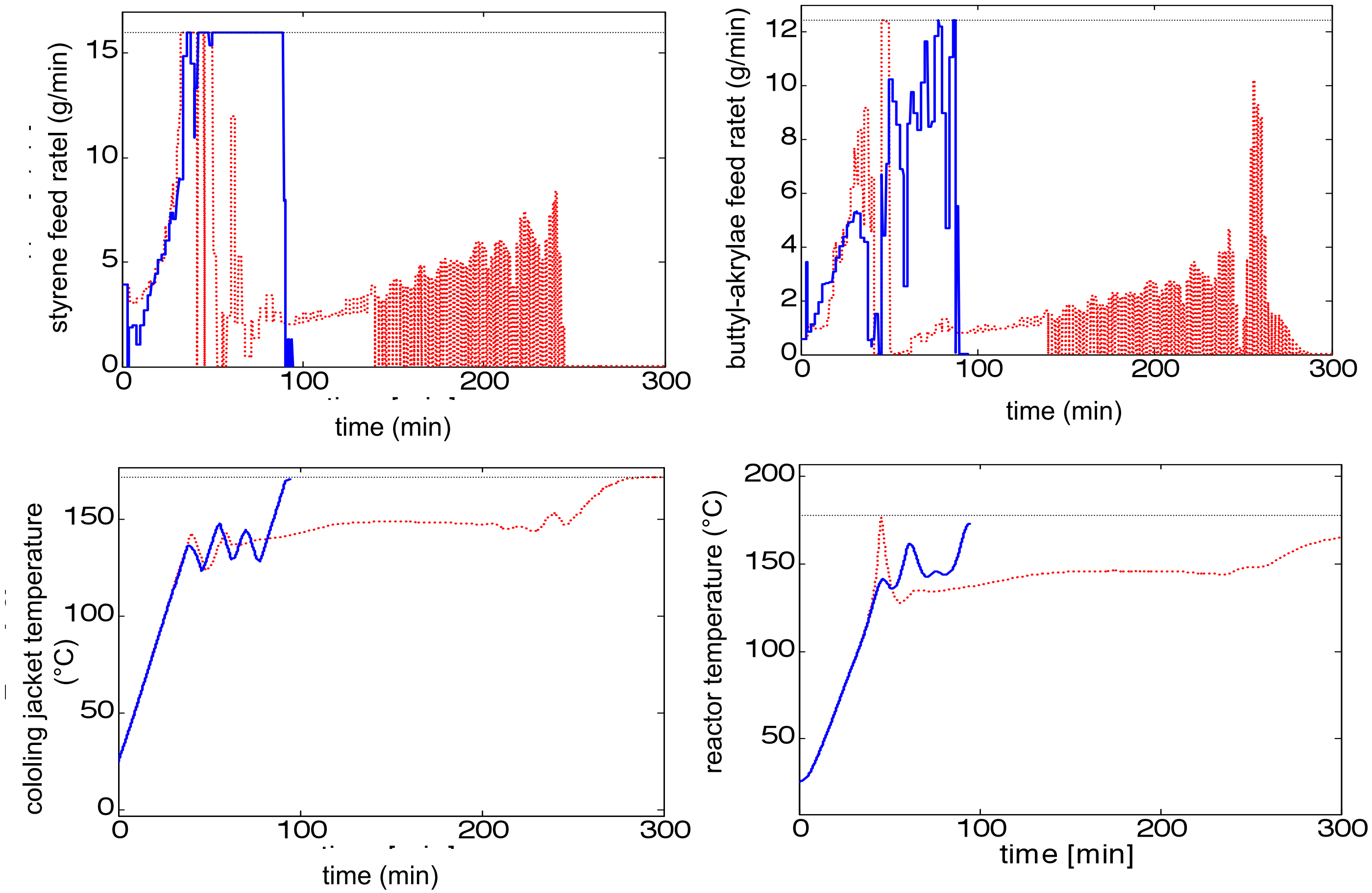
limited modeling effort, significant model uncertainty



scope for dynamic real-time optimization in industrial environment?

Experimental Results

Dynamic real-time optimization with nominal model and multi-model



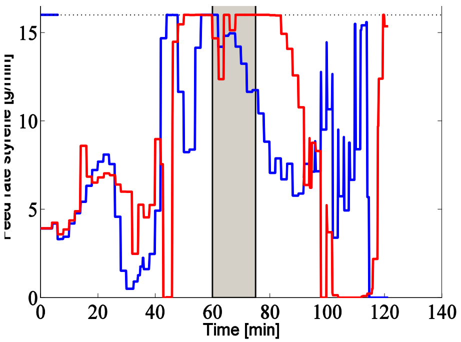
2 min sampling time

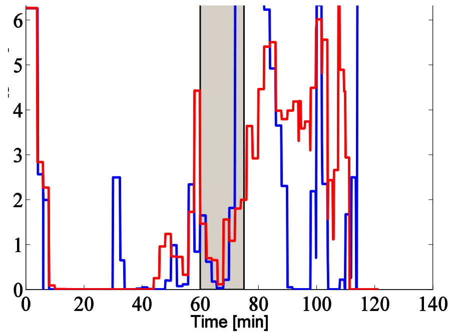
- **nominal does not work due to model uncertainty**
- **robustification with simultaneous optimization of nominal and worst-case model**
- **multi-model strategy meets quality specs and reduces batch time significantly**

Recovery after Feed Pump Failure

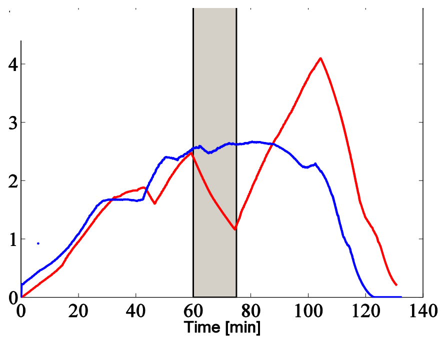
- Scenario **without** / with **simulated** styrene pump failure
  - Styrene pump switched-off for 15 minutes

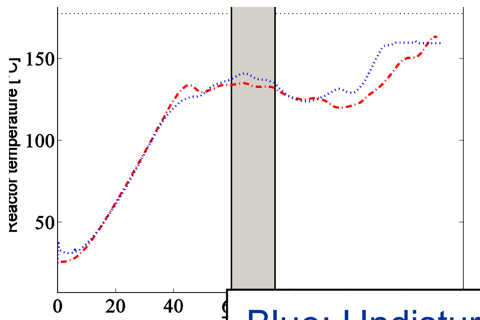
Control signals of the optimizer to the process





Measurements of the process





Blue: Undisturbed scenario
Red: With disturbance
Grey: Actuator fail

On-Site and Software Implementation